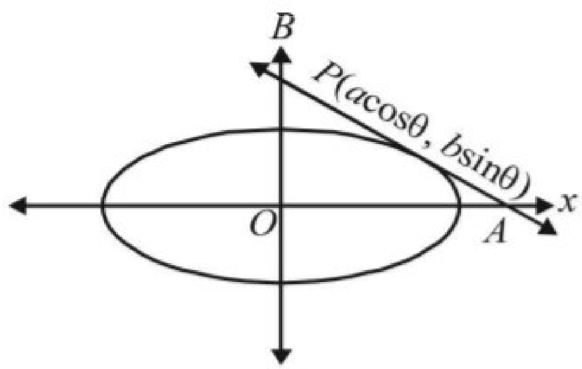
3. The minimum area of triangle formed by the tangent to the $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & coordinate axes is ***(2005S)***

(a) ab sq. units (b) $\frac{a^2+b^2}{2}$ sq. units

(c) $\frac{(a+b)^2}{2}$ sq. units (d) $\frac{a^2+ab+b^2}{3}$ sq. units

Solution: -

3. **(a)** Any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at $P(a\cos\theta, b\sin\theta)$ is $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$



2

It meets co-ordinate axes at $A(a\sec\theta, 0)$ and $B(0, b\operatorname{cosec}\theta)$

$\therefore$ Area of $\Delta OAB = \frac{1}{2}\times a\sec\theta\times b\operatorname{cosec}\theta$

$\Rightarrow \Delta = \frac{ab}{\sin 2\theta}$

For Δ to be min, $\sin 2\theta$ should be max. and we know max value of $\sin 2\theta = 1$

$\therefore \Delta_{max} = ab$ sq. units.