Q . The equation of the ellipse, whose focus is $(1, -1)$, the directrix the line $x - y - 3 = 0$ and eccentricity $\dfrac{1}{2}$, is

(a) $7x^2 + 2xy + 7y^2 - 10x + 10y + 7 = 0$
(b) $7x^2 + 2xy + 7y^2 + 7 = 0$
(c) $7x^2 + 2xy + 7y^2 + 10x - 10y - 7 = 0$
(d) None of the above

Sol. Given that focus of the ellipse is $(1, -1)$ and the equation of the directrix is $x - y - 3 = 0$ and $e = \dfrac{1}{2}$.

Let $P(x, y)$ by any point on the ellipse.

$$\therefore \frac{PF}{\text{Distance of the point } P \text{ from the directrix}} = e$$

$$= \frac{\sqrt{(x-1)^2 + (y+1)^2}}{\left| \dfrac{x - y - 3}{\sqrt{(1)^2 + (-1)^2}} \right|} = \frac{1}{2}$$

$$\Rightarrow 2\sqrt{x^2 + 1 - 2x + y^2 + 1 + 2y} = \left| \frac{x - y - 3}{\sqrt{2}} \right|$$

Squaring both sides, we have

$$\Rightarrow 4(x^2 + y^2 - 2x + 2y + 2) = \frac{x^2 + y^2 + 9 - 2xy + 6y - 6x}{2}$$

$\Rightarrow 8x^2 + 8y^2 - 16x + 16y + 16 = x^2 + y^2 - 2xy + 6y - 6x + 9$
$\Rightarrow 7x^2 + 7y^2 + 2xy - 10x + 10y + 7 = 0$
Hence, the correct option is (a).