(iii) The nuclear force between neutron-neutron, proton-neutron and proton-proton is approximately the same. The nuclear force does not depend on the electric charge.

Unlike Coulomb's law or the Newton's law of gravitation there is no simple mathematical form of the nuclear force.

13.6 Radioactivity

A. H. Becquerel discovered radioactivity in 1896 purely by accident. While studying the fluorescence and phosphorescence of compounds irradiated with visible light, Becquerel observed an interesting phenomenon. After illuminating some pieces of uranium-potassium sulphate with visible light, he wrapped them in black paper and separated the package from a photographic plate by a piece of silver. When, after several hours of exposure, the photographic plate was developed, it showed blackening due to something that must have been emitted by the compound and was able to penetrate both black paper and the silver.

Experiments performed subsequently showed that radioactivity was a nuclear phenomenon in which an unstable nucleus undergoes a decay. This is referred to as *radioactive decay*. Three types of radioactive decay occur in nature :

(i) α-decay in which a helium nucleus ${}^{4}_{2}\mathrm{He}$ is emitted;

(ii) β-decay in which electrons or positrons (particles with the same mass as electrons, but with a charge exactly opposite to that of electron) are emitted;

(iii) γ-decay in which high energy (hundreds of keV or more) photons are emitted.

Each of these decay will be considered in subsequent sub-sections.

13.6.1 Law of radioactive decay

In any radioactive sample, which undergoes α, β or γ-decay, it is found that the number of nuclei undergoing the decay per unit time is proportional to the total number of nuclei in the sample. If N is the number of nuclei in the sample and ΔN undergo decay in time Δt then

$$\frac{\Delta N}{\Delta t} \propto N$$

or, $\Delta N/\Delta t = \lambda N$, (13.10)

where λ is called the radioactive *decay constant* or *disintegration constant*.

The change in the number of nuclei in the sample* is $dN = -\Delta N$ in time Δt. Thus the rate of change of N is (in the limit $\Delta t \to 0$)

$$\frac{dN}{dt} = -\lambda N$$

* ΔN is the number of nuclei that decay, and hence is always positive. dN is the change in N, which may have either sign. Here it is negative, because out of original N nuclei, ΔN have decayed, leaving $(N–\Delta N)$ nuclei.

or, $\frac{dN}{N} = -\lambda dt$

Now, integrating both sides of the above equation,we get,

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_{t_0}^{t} dt \tag{13.11}$$

or, $\ln N - \ln N_0 = -\lambda (t - t_0)$ (13.12)

Here N_0 is the number of radioactive nuclei in the sample at some arbitrary time t_0 and N is the number of radioactive nuclei at any subsequent time t. Setting t_0 = 0 and rearranging Eq. (13.12) gives us

$$\ln \frac{N}{N_0} = -\lambda t \tag{13.13}$$

which gives

$$N(t) = N_0 \, e^{-\lambda t} \tag{13.14}$$

Note, for example, the light bulbs follow no such exponential decay law. If we test 1000 bulbs for their life (time span before they burn out or fuse), we expect that they will 'decay' (that is, burn out) at more or less the same time. The decay of radionuclides follows quite a different law, the *law of radioactive decay* represented by Eq. (13.14).

The total decay rate R of a sample is the number of nuclei disintegrating per unit time. Suppose in a time interval dt, the decay count measured is ΔN. Then $dN = -\Delta N$.

The positive quantity R is then defined as

$$R = -\frac{dN}{dt}$$

Differentiating Eq. (13.14), we get

$$R = \lambda N_0 \, e^{-\lambda t}$$

or, $R = R_0 \, e^{-\lambda t}$ (13.15)

This is equivalant to the law of radioactivity decay, since you can integrate Eq. (13.15) to get back Eq. (13.14). Clearly, $R_0 = \lambda N_0$ is the decay rate at $t = 0$. The decay rate R at a certain time t and the number of undecayed nuclei N at the same time are related by

$$R = \lambda N \tag{13.16}$$

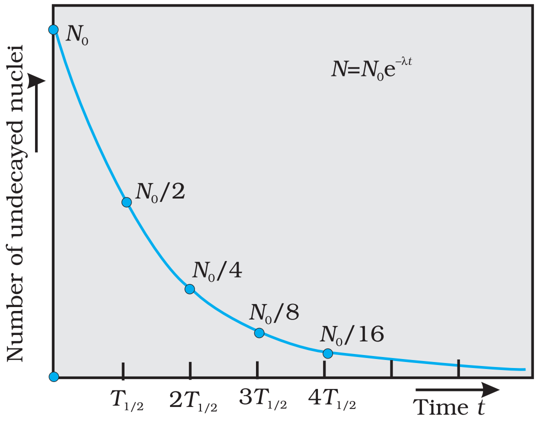

FIGURE 13.3 Exponential decay of a radioactive species. After a lapse of $T_{1/2}$, population of the given species drops by a factor of 2.

The decay rate of a sample, rather than the number of radioactive nuclei, is a more direct experimentally measurable quantity and is given a specific name: *activity*. The SI unit for activity is becquerel, named after the discoverer of radioactivity, Henry Becquerel.

1 becquerel is simply equal to 1 disintegration or decay per second. There is also another unit named "curie" that is widely used and is related to the SI unit as:

$$1 \text{ curie} = 1 \text{ Ci} = 3.7 \times 10^{10} \text{ decays per second}$$
$$= 3.7 \times 10^{10} \text{ Bq}$$

MARIE SKLODOWSKA CURIE (1867-1934)

Marie Sklodowska Curie (1867-1934) Born in Poland. She is recognised both as a physicist and as a chemist. The discovery of radioactivity by Henri Becquerel in 1896 inspired Marie and her husband Pierre Curie in their researches and analyses which led to the isolation of radium and polonium elements. She was the first person to be awarded two Nobel Prizes- for Physics in 1903 and for Chemistry in 1911.

Different radionuclides differ greatly in their rate of decay. A common way to characterize this feature is through the notion of *half-life*. Half-life of a radionuclide (denoted by $T_{1/2}$) is the time it takes for a sample that has initially, say N_0 radionuclei to reduce to $N_0/2$. Putting $N = N_0/2$ and $t = T_{1/2}$ in Eq. (13.14), we get

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \tag{13.17}$$

Clearly if N_0 reduces to half its value in time $T_{1/2}$, R_0 will also reduce to half its value in the same time according to Eq. (13.16).

Another related measure is the *average* or *mean life* τ. This again can be obtained from Eq. (13.14). The number of nuclei which decay in the time interval t to $t + \Delta t$ is $R(t)\Delta t$ $(= \lambda N_0 e^{-\lambda t} \Delta t)$. Each of them has lived for time t. Thus the total life of all these nuclei would be $t \lambda N_0 e^{-\lambda t} \Delta t$. It is clear that some nuclei may live for a short time while others may live longer. Therefore to obtain the mean life, we have to sum (or integrate) this expression over all times from 0 to ∞, and divide by the total number N_0 of nuclei at $t = 0$. Thus,

$$\tau = \frac{\lambda N_0 \int_0^\infty t e^{-\lambda t} dt}{N_0} = \lambda \int_0^\infty t e^{-\lambda t} dt$$

One can show by performing this integral that

$\tau = 1/\lambda$

We summarise these results with the following:

$$T_{1/2} = \frac{\ln 2}{\lambda} = \tau \ln 2 \tag{13.18}$$

Radioactive elements (e.g., tritium, plutonium) which are short-lived i.e., have half-lives much less than the age of the universe ($\sim$ 15 billion years) have obviously decayed long ago and are not found in nature. They can, however, be produced artificially in nuclear reactions.

EXAMPLE 13.4

Example 13.4 The half-life of $^{238}_{92}U$ undergoing α-decay is 4.5×10^9 years. What is the activity of 1g sample of $^{238}_{92}U$?

Solution

$T_{1/2} = 4.5 \times 10^9$ y
$= 4.5 \times 10^9$ y x 3.16 x 10^7 s/y
$= 1.42 \times 10^{17}$s