Tips & Tricks for Trigonometry

Trigonometric Equations :→

Ways to solve trigonometric equations :→

Type 1 Use of factorizations

Eg $(2\sin x - \cos x)(1+\cos x) = \sin^2 x$

$\Rightarrow (2\sin x - \cos x)\underbrace{(1+\cos x)}_{\text{common}} - \underbrace{(1+\cos x)}(1-\cos x) = 0$

Type 2 Reduce the trigonometric equations into quadratic equations.

Eg $2\cos^2 x + 4\cos x = 3\sin^2 x$ ← $1-\cos^2 x$ $\quad \cos x = t$

Type 3 Use of Sum or difference of trigonometric ratios (Convert them into product)

$\sin C \pm \sin D \qquad \cos C \pm \cos D$

Type 4. Convert product of trigonometric ratios into the sum or difference of their angles.

$2\sin A \cos B = \sin(A+B) + \sin(A-B)$

Type 5. $a\sin x + b\cos x = c$

dividing by $\sqrt{a^2+b^2}$ on both sides

$$\underbrace{\frac{a}{\sqrt{a^2+b^2}}}_{\cos\alpha}\sin x + \underbrace{\frac{b}{\sqrt{a^2+b^2}}}_{\sin\alpha}\cos x = \frac{c}{\sqrt{a^2+b^2}} \rightarrow \text{Eq becomes}$$

$$\sin x\cos\alpha + \cos x\sin\alpha = \frac{c}{\sqrt{a^2+b^2}}$$

$$\Rightarrow \sin(x+\alpha) = \frac{c}{\sqrt{a^2+b^2}}$$

<u>Another way</u> $\quad \sin x = \dfrac{2\tan x/2}{1+\tan^2 x/2} \qquad \cos x = \dfrac{1-\tan^2 x/2}{1+\tan^2 x/2}$

Quadratic in $\tan\frac{x}{2}$.

<u>Type 6.</u> Sometimes the trigonometric equations can be solved using boundness of trigonometric ratios $\sin x$ & $\cos x$.

$$\boxed{\begin{array}{l} -1 \le \sin x \le 1 \\ -1 \le \cos x \le 1 \end{array}}$$

<u>Eg</u> $\quad \sin\left(\frac{5\theta}{4}\right) + \cos(\theta) = 2$

Here $\sin\left(\frac{5\theta}{4}\right) \in [-1, 1]$, Also $\cos(\theta) \in [-1, 1]$. So,

Max Value of L.H.S. $= 2 =$ RHS.

This is possible only when $\sin\left(\frac{5\theta}{4}\right) = 1$ as well as $\cos(\theta) = 1$

<u>Type 7.</u> $P(\sin x \pm \cos x, \sin x \cos x) = 0$; can be solved using $\sin x \pm \cos x = t$

Polynomial

<u>Eg</u> $\quad \sin x + \cos x = 1 + \sin x \cos x$

Assume $\sin x + \cos x = t$

<u>Sq</u> $(\sin x + \cos x)^2 = t^2$

$\Rightarrow \quad 1 + 2\sin x \cos x = t^2$

$\Rightarrow \quad \sin x \cos x = \dfrac{t^2 - 1}{2}$

<u>Imp</u>

- While solving trigonometric equations, we must be careful against the danger of loosing roots while we cancel a common factor.

Whenever we cancel a common factor, make a separate case when that common factor can be equal to zero.

- Avoid Squaring (if possible) because there is a danger of additional roots.

For Eg $\quad \sin\theta + \cos\theta = 1 \longrightarrow (\sin\theta + \cos\theta)^2 = 1$

Sq $\quad 1 + \sin 2\theta = 1$

get back

$\boxed{\sin\theta + \cos\theta = 1}$ or $\boxed{\sin\theta + \cos\theta = -1}$

If squaring, make sure to satisfy the solutions you get by substituting back in the original equation. This is will omit extra solutions.

- Whenever Equation involves $\tan x$ or $\sec x$, always take $\cos x \neq 0$. Similarly if equation involves $\cot x$ or $\operatorname{cosec} x$ take $\sin x \neq 0$.

- Take care of domain always.

Eg $\quad \log(f(\theta)) \Rightarrow \boxed{f(\theta) > 0} \qquad \sqrt{f(\theta)} \quad \boxed{f(\theta) > 0}$

- $\sqrt{f(\theta)} \rightarrow$ always positive

$\sqrt{\sin^2\theta} = |\sin\theta|$ not $\pm \sin\theta$

- General advise $\rightarrow$ always crosscheck the solutions we have got.

* If $\tan A + \tan B + \tan C = 0$

$\Rightarrow \quad A + B + C = n\pi$ (general soln)

* If $\tan A \tan B + \tan B \tan C + \tan C \tan A = 0$

$\Rightarrow \quad A + B + C = \pm(2n+1)\pi$ (general soln)

$\Rightarrow \quad A+B+C = \pm(2n+1)\frac{\pi}{2}$ (general sol)

- Take care of denominator never becoming zero.