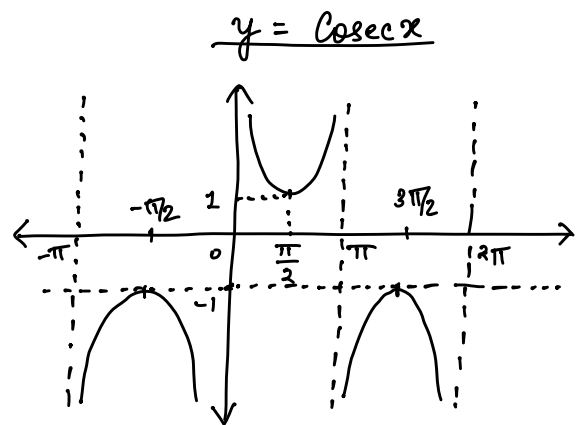
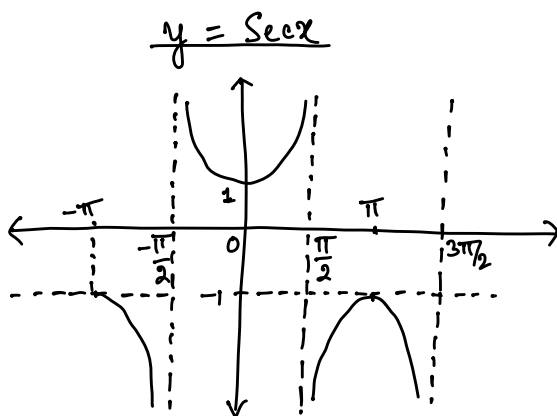
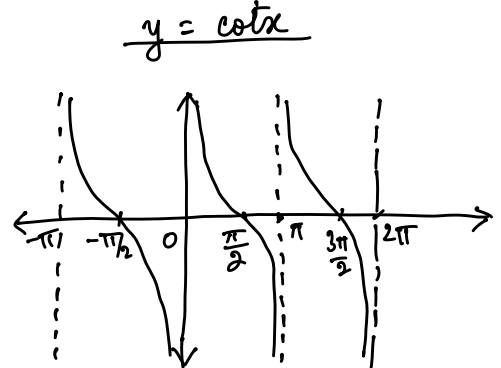
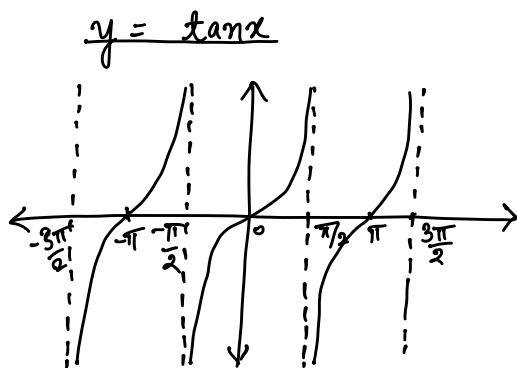
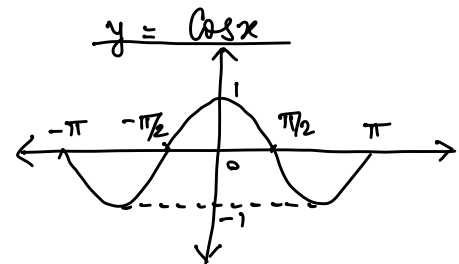
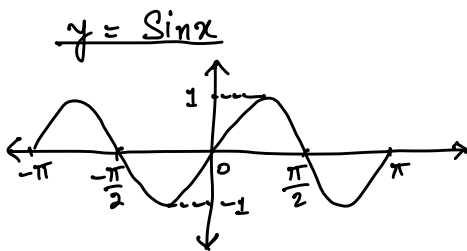


# Trigonometric Functions

1.) 1 degree =  $\left(\frac{\pi}{180}\right)$  radians

2.) Graphs  $\rightarrow$



3.) Important Formulas  $\rightarrow$

a.)  $\sin(-x) = -\sin x$

$$b.) \cos(-x) = \cos x$$

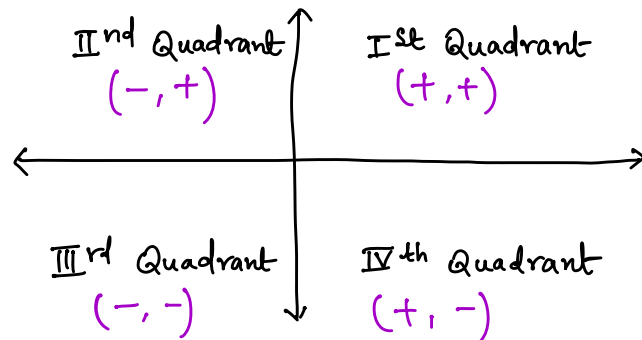
$$c.) \tan(-x) = -\tan x$$

$$d.) \sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$e.) \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$f.) \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$g.) \cos(A-B) = \cos A \cos B + \sin A \sin B$$



• x coordinate represents Sine function

• y coordinate represents Cosine function

- a) In 1<sup>st</sup> Quadrant, both Sine & Cosine are (+)ve.  
 b) In 2<sup>nd</sup> Quadrant, Sine is (-)ve, Cosine is (+)ve.  
 c) In 3<sup>rd</sup> Quadrant, both Sine & Cosine are (-)ve.  
 d) In 4<sup>th</sup> Quadrant, Sine is (+)ve, Cosine is (-)ve.

$$\cos\left(\frac{\pi}{2}-x\right) = \sin x ; \quad \sin\left(\frac{\pi}{2}-x\right) = \cos x ; \quad \cos\left(\frac{\pi}{2}+x\right) = -\sin x ;$$

$$\sin\left(\frac{\pi}{2}+x\right) = \cos x ; \quad \cos(\pi-x) = -\cos x ; \quad \sin(\pi-x) = \sin x ;$$

$$\cos(2\pi-x) = \cos x ; \quad \sin(2\pi-x) = -\sin x ;$$

$$* \quad \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \quad A+B \neq (2n+1)\frac{\pi}{2}$$

$$* \quad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \quad A-B \neq (2n+1)\frac{\pi}{2}$$

$$* \overset{\text{vimp}}{\text{vimp}} \quad \sin(2x) = 2 \sin(x) \cos(x) = \frac{2 \tan x}{1 + \tan^2 x}$$

$$* \overset{\text{vimp}}{\text{vimp}} \quad \cos(2x) = \cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$* \text{Vimp} \quad \tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$$

$$* \begin{cases} \sin(3x) = 3 \sin x - 4 \sin^3 x \\ * \text{Vimp} \quad \cos(3x) = 4 \cos^3 x - 3 \cos x \\ * \quad \tan(3x) = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} \end{cases}$$

$$* \left\{ \begin{array}{l} \cos C + \cos D = 2 \cos \left( \frac{C+D}{2} \right) \cos \left( \frac{C-D}{2} \right) \\ \cos C - \cos D = -2 \sin \left( \frac{C+D}{2} \right) \sin \left( \frac{C-D}{2} \right) \\ \sin C + \sin D = 2 \sin \left( \frac{C+D}{2} \right) \cos \left( \frac{C-D}{2} \right) \\ \sin C - \sin D = 2 \cos \left( \frac{C+D}{2} \right) \sin \left( \frac{C-D}{2} \right) \end{array} \right.$$

Helpful

- $2 \cos x \cos y = \cos(x+y) + \cos(x-y)$
- $-2 \sin x \sin y = \cos(x+y) - \cos(x-y)$
- $2 \sin x \cos y = \sin(x+y) + \sin(x-y)$
- $2 \cos x \sin y = \sin(x+y) - \sin(x-y)$

$$\bullet \sin^2 A - \sin^2 B = \sin(A+B) \sin(A-B) = \cos^2(B) - \cos^2(A)$$

$$\bullet \cos^2 A - \cos^2 B = \cos(A+B) \cos(A-B)$$

imp

$$* \tan(A+B+C) = \frac{\tan A + \tan B + \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$$

$$\text{if } A+B+C = \pi \Rightarrow \boxed{\tan A + \tan B + \tan C = 0} \quad \text{imp}$$

\* Sine & Cosine Product Series with Angles in A.P. (Very Useful)

$$\sin(\theta) \sin(\theta+\beta) \sin(\theta+2\beta) \dots \sin(\theta+(n-1)\beta) = \frac{\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)} \sin\left(\theta + \frac{(n-1)\beta}{2}\right)$$

---

$$\cos(\theta) \cos(\theta+\beta) \cos(\theta+2\beta) \dots \cos(\theta+(n-1)\beta) = \frac{\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)} \cos\left(\theta + \frac{(n-1)\beta}{2}\right)$$

---

For tan,

$$\tan(\theta) \tan(\theta+\beta) \tan(\theta+2\beta) \dots \tan(\theta+(n-1)\beta) = \tan\left(\theta + \frac{(n-1)\beta}{2}\right)$$

---

Some imp values :->

1)  $\sin(18^\circ) = \cos(72^\circ) = \sin\left(\frac{\pi}{10}\right) = \frac{\sqrt{5}-1}{4}$

2)  $\cos(36^\circ) = \sin(54^\circ) = \cos\left(\frac{\pi}{5}\right) = \frac{\sqrt{5}+1}{4}$

3)  $\sin(15^\circ) = \cos(75^\circ) = \sin\left(\frac{\pi}{12}\right) = \frac{\sqrt{3}-1}{2\sqrt{2}}$

4)  $\cos(15^\circ) = \sin(75^\circ) = \cos\left(\frac{\pi}{12}\right) = \frac{\sqrt{3}+1}{2\sqrt{2}}$

5)  $\tan\left(\frac{\pi}{12}\right) = 2-\sqrt{3} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \cot\left(\frac{5\pi}{12}\right)$

6)  $\tan\left(\frac{5\pi}{12}\right) = 2+\sqrt{3} = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \cot\left(\frac{\pi}{12}\right)$

7)  $\tan(22.5^\circ) = \sqrt{2}-1$

8)  $\tan(67.5^\circ) = \cot(22.5^\circ) = \sqrt{2}+1$

## Minimum and Maximum Values of Trigonometric Equations

$$a) a \cos \theta + b \sin \theta \rightarrow \left[ -\sqrt{a^2+b^2}, \sqrt{a^2+b^2} \right]$$

$\downarrow$                        $\downarrow$   
Min. Value              Max Value

$$b) \text{ Min value of } a^2 \tan^2 \theta + b^2 \cot^2 \theta = 2ab \quad a, b > 0$$

## Other Imp Results

$$a) \sin(\theta) \sin(60^\circ - \theta) \sin(60^\circ + \theta) = \frac{1}{4} \sin(3\theta)$$

$$b) \cos(\theta) \cos(60^\circ - \theta) \cos(60^\circ + \theta) = \frac{1}{4} \cos(3\theta)$$

$$c) \tan(\theta) \tan(60^\circ - \theta) \tan(60^\circ + \theta) = \tan(3\theta)$$

} imp

$$d) \sin^2(\theta) + \sin^2(60^\circ + \theta) + \sin^2(60^\circ - \theta) = \frac{3}{2} = \cos^2(\theta) + \cos^2(60^\circ + \theta) + \cos^2(60^\circ - \theta)$$

$$e) \cos \theta \cdot \cos 2\theta \cdot \cos 4\theta \dots \cos(2^{n-1}\theta) = \frac{\sin(2^n \theta)}{2^n \sin(\theta)} \quad (\text{imp})$$

$$f) \cot A - \tan A = 2 \cot 2A \quad (\text{imp})$$

## Conditional Identities

$$\text{If } A + B + C = \pi$$

$$a) \tan A + \tan B + \tan C = \tan A \tan B \tan C$$

$$b) \cot A \cot B + \cot B \cot C + \cot C \cot A = 1$$

$$c) \sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$$

$$d) \cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$$

$$e) \sum \frac{\tan A}{2} \frac{\tan B}{2} = 1$$

$$f) \sum \frac{\cot A}{2} \frac{\cot B}{2} = \frac{\cot A}{2} \frac{\cot B}{2} \frac{\cot C}{2}$$

$$g.) \quad \sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$$

$$h.) \quad \cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

### General Solutions

- $\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha \quad \alpha \in [-\pi/2, \pi/2], n \in \mathbb{I}$
- $\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha \quad \alpha \in [0, \pi], n \in \mathbb{I}$
- $\tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha \quad \alpha \in (-\pi/2, \pi/2), n \in \mathbb{I}$
- $\sin^2 \theta = \sin^2 \alpha \Rightarrow \theta = n\pi \pm \alpha, n \in \mathbb{I}$
- $\cos^2 \theta = \cos^2 \alpha \Rightarrow \theta = n\pi \pm \alpha, n \in \mathbb{I}$
- $\tan^2 \theta = \tan^2 \alpha \Rightarrow \theta = n\pi \pm \alpha, n \in \mathbb{I}$