Equation of Plane in Normal Form

(a) Vector Form

> *The vector equation of a plane normal to unit vector $\hat{n}$ and at a distance d from the origin is*
>
> $\vec{r}\,.\,\hat{n} = d$

Remark 1 : The vector equation of ON is $\vec{r} = \vec{0} + \lambda\,\hat{n}$ and the position vector of N is $d\hat{n}$ as it is at a distance d from the origin from the origin O.

(b) Cartesian Form

> *If l, m, n are direction cosines of the normal to a given plane which is at a distance p from the origin, then the equation of the plane is*
>
> $lx + my + nz = p$

Note : The equation $\vec{r}\,.\,\vec{n}$ = d is in normal form if $\vec{n}$ is a unit vector and in such a case d on the right hand side denotes the distance of the plane from the origin. If $\vec{n}$ is not a unit vector, then to reduce the equation $\vec{r}\,.\,\vec{n}$ = d to normal form divide both sides by | $\vec{n}$ | to obtain

$$\vec{r}\,.\,\frac{\vec{n}}{|\vec{n}|} = \frac{d}{|\vec{n}|} \implies \vec{r}\,.\,\hat{n} = \frac{d}{|\vec{n}|}$$

Equation of Plane Passing Through Intersection of Two Planes

(a) Vector Form

The equation of a plane passing through the intersection of the planes $\vec{r}.\overrightarrow{n_1} = d_1$ and $\vec{r}.\overrightarrow{n_2} = d_2$ is given by

> $(\vec{r}.\overrightarrow{n_1} - d_1) + \lambda(\vec{r}.\overrightarrow{n_2} - d_2) = 0$
>
> *or,* $\vec{r}.(\overrightarrow{n_1} + \lambda\overrightarrow{n_2}) = d_1 + \lambda d_2,$
>
> *where λ is an arbitrary constant.*

(b) Cartesian Form

The equation of a plane passing through the intersection of planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is

> $(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$,
>
> *where λ is a constant.*