

### **MULTIPLICATION OF TWO DETERMINANTS :**

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} l_1 & m_1 \\ l_2 & m_2 \end{vmatrix} = \begin{vmatrix} a_1 l_1 + b_1 l_2 & a_1 m_1 + b_1 m_2 \\ a_2 l_1 + b_2 l_2 & a_2 m_1 + b_2 m_2 \end{vmatrix}$$

Similarly two determinants of order three are multiplied.

- (a)** Here we have multiplied row by column. We can also multiply row by row, column by row and column by column.
- (b)** If  $D_1$  is the determinant formed by replacing the elements of determinant  $D$  of order  $n$  by their corresponding cofactors then  $D_1 = D^{n-1}$

## SPECIAL DETERMINANTS :

### (a) Cyclic Determinant :

The elements of the rows (or columns) are in cyclic arrangement.

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3 + b^3 + c^3 - 3abc) = -(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$$

$$= -\frac{1}{2}(a + b + c) \times \{(a - b)^2 + (b - c)^2 + (c - a)^2\}$$

$$= -(a + b + c)(a + b\omega + c\omega^2)(a + b\omega^2 + c\omega), \text{ where } \omega, \omega^2 \text{ are cube roots of unity}$$

### (b) Other Important Determinants :

$$(i) \begin{vmatrix} 0 & b & -c \\ -b & 0 & a \\ c & -a & 0 \end{vmatrix} = 0$$

$$(ii) \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ac & ab \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a - b)(b - c)(c - a)$$

$$(iii) \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(a + b + c)$$

$$(iv) \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(ab + bc + ca)$$

$$(v) \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^4 & b^4 & c^4 \end{vmatrix} = (a - b)(b - c)(c - a)(a^2 + b^2 + c^2 - ab - bc - ca)$$

(i) The value of the determinant  $\begin{vmatrix} ka & k^2 + a^2 & 1 \\ kb & k^2 + b^2 & 1 \\ kc & k^2 + c^2 & 1 \end{vmatrix}$  is

(A)  $k(a + b)(b + c)(c + a)$

(B)  $kabc(a^2 + b^2 + c^2)$

(C)  $k(a - b)(b - c)(c - a)$

(D)  $k(a + b - c)(b + c - a)(c + a - b)$

(ii) Find the value of the determinant  $\begin{vmatrix} a^2 + b^2 & a^2 - c^2 & a^2 - c^2 \\ -a^2 & 0 & c^2 - a^2 \\ b^2 & -c^2 & b^2 \end{vmatrix}$ .

(iii) Prove that  $\begin{vmatrix} a & b & c \\ bc & ca & ab \\ b + c & c + a & a + b \end{vmatrix} = (a + b + c)(a - b)(b - c)(c - a)$ .