7.19 to 16.10 : vector normal form and example

A plane is at a distance of $\frac{9}{\sqrt{38}}$ from the origin O. From the origin, its normal vector is given by $5\hat{i} + 3\hat{j} - 2\hat{k}$.

What is the vector equation for the plane?

Solution:

Let the normal vector be:

$\vec{n} = 5\hat{i} + 3\hat{j} - 2\hat{k}$

We now find the unit vector for the normal vector. It can be given by:

$\hat{n} = \frac{\vec{n}}{|\vec{n}|}$

$\hat{n} = \frac{5\hat{i}+3\hat{j}-2\hat{k}}{\sqrt{25+9+4}}$

$\hat{n} = \frac{5\hat{i}+3\hat{j}-2\hat{k}}{\sqrt{38}}$

So, the required equation of the plane can be given by substituting it in the vector equation is:

$\vec{r}.\left(\frac{5}{\sqrt{38}}\hat{i} + \frac{3}{\sqrt{38}}\hat{j} + \frac{-2}{\sqrt{38}}\hat{k}\right) = \frac{9}{\sqrt{38}}$

16.11 to 21.10 : cartesian normal form and example

Example : Find the vector equation of a plane which is at a distance of 8 units from the origin and which is normal to the vector $2\hat{i} + \hat{j} + 2\hat{k}$.

Solution : Here, d = 8 and $\vec{n} = 2\hat{i} + \hat{j} + 2\hat{k}$

$\therefore \hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{2\hat{i}+\hat{j}+2\hat{k}}{\sqrt{4+1+4}}$

$= \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$

Hence, the required equation of the plane is

$\vec{r}.(\frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}) = 8$

[By using $\vec{r}.\hat{n} = d$]

or, $\vec{r}.(2\hat{i} + \hat{j} + 2\hat{k}) = 24$

21.11to 24.45 : vector form of plane through 3non-collinear points

Here, we have to find the equation of the plane passing through the points A (0, - 1, 0), B (1, 1, 1) and C (3, 3, 0)

Here, $x_1 = 0, y_1 = -1, z_1 = 0, x_2 = 1, y_2 = 1, z_2 = 1, x_3 = 3, y_3 = 3$ and $z_3 = 0$.

As we know that, equation of the plane in Cartesian form passing through three non collinear points (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) is given by:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-0 & y+1 & z-0 \\ 1 & 2 & 1 \\ 3 & 4 & 0 \end{vmatrix} = 0$$

$\Rightarrow (x - 0) \times (0 - 4) - (y + 1) \times (0 - 3) + (z - 0) \times (4 - 6) = 0$

$\Rightarrow -4x + 3y + 3 - 2z = 0$

$\Rightarrow 4x - 3y + 2z - 3 = 0$

So, the equation of the required plane is $4x - 3y + 2z - 3 = 0$

37.48to 47.44 : equation of plane through intersection of two planes and example

Example : Find the equation of the plane containing the line of intersection of the planes $x + y + z - 6 = 0$ and $2x + 3y + 4z + 5 = 0$ and passing through the point (1, 1, 1),

Solution : The equation of the plane through the line of intersection of the given planes is

$(x + y + z - 6) + \lambda(2x + 3y + 4z + 5) = 0$ ....(i)

If (i) passes through (1, 1, 1), then

$-3 + 14\lambda = 0 \Longrightarrow \lambda = 3/14$

Putting $\lambda = 3/14$ in equation (i), we obtain the equation of the required plane as

$(x + y + z - 6) + \frac{3}{14}(2x + 3y + 4z + 5) = 0$

or, $20x + 23y + 26z - 69 = 0$.