

LCR circuit

(1)

$$L \frac{dI}{dt} + IR + \frac{Q}{C} = V_m \sin \omega t$$

$$I = \frac{dQ}{dt}$$

$$L \frac{d^2 I}{dt^2} + R \frac{dI}{dt} + \frac{I}{C} = V_m \omega \cos \omega t$$

$$I(t) = I_m \sin(\omega t + \phi)$$

↳ Amount of phase by which current leads the voltage

$$I_m = \frac{V_m}{Z} ; Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$\tan \phi = \frac{X_C - X_L}{R} ; X_C = \frac{1}{\omega C} , X_L = \omega L$$

Maximum current I_m has a frequency dependence

I_m has a maximum value as function of ω .

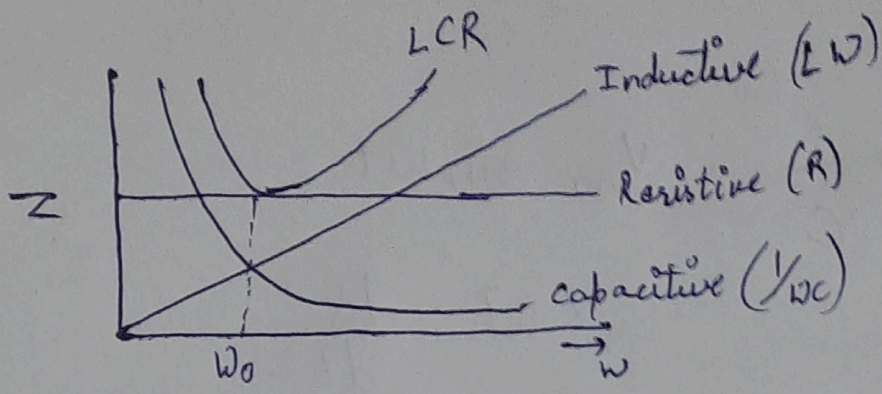
$$I_m = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

I_m has a maximum as a function of ω when

$$X_C = X_L \Rightarrow \omega L = \frac{1}{\omega C}$$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}} \text{ (Resonance)}$$

$$\boxed{I_m^{\max} = \frac{V_m}{R}}$$



At $\omega = \omega_0$, the circuit absorbs maximum power

$$I_m \propto \frac{1}{Z}$$

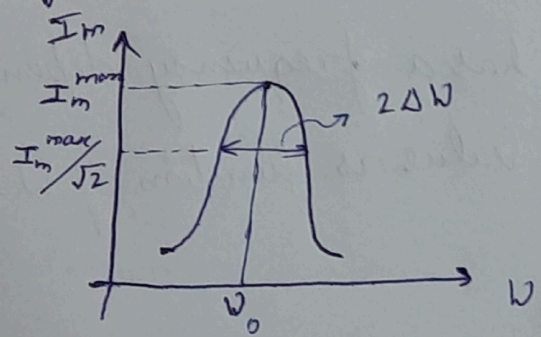
$$I_m^2 Z \propto \frac{1}{Z}$$

Maximum power

$\Rightarrow Z$ is minimum

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$

Half power points



$I_m = \frac{I_m^{\max}}{\sqrt{2}}$; The power absorbed is half the possible maximum I_m^{\max}

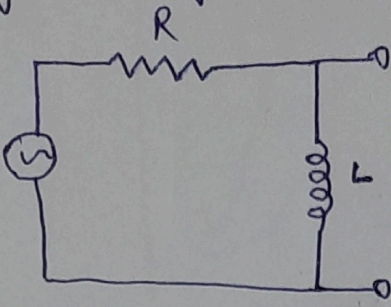
full width at half max $2\Delta\omega$ bandwidth

$$\Delta\omega = \frac{R}{2L} \quad ; \quad \text{Band width} \quad BW = 2\Delta\omega = \frac{R}{L}$$

Quality factor

$$Q = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R}$$

Typical circuit application $Q \sim 10$ to 100

High pass filter

$$I_m = \frac{V_{in}^m}{\sqrt{R^2 + L^2 \omega^2}}$$

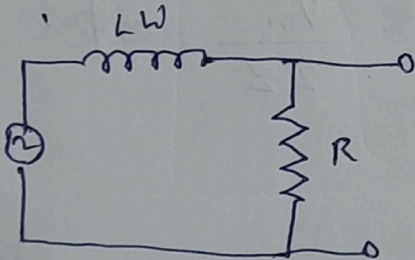
$$V_{out} = \frac{V_{in}^m}{\sqrt{R^2 + L^2 \omega^2}} \times L\omega$$

$$= V_{in}^m \cdot \frac{L\omega}{L\omega \left(1 + \frac{R^2}{L^2 \omega^2}\right)^{1/2}}$$

$$\approx V_{in}^m \left[1 - \frac{1}{2} \frac{R^2}{L^2 \omega^2}\right]$$

As ω increases $V_{out} \rightarrow V_{in}$

$\omega \rightarrow 0$ $V_{out} = 0$

Low pass filter

$$V_{out} = \frac{V_{in} R}{\sqrt{R^2 + L^2 \omega^2}}$$

$$= \frac{V_{in} R}{L\omega \left[1 + \frac{R^2}{L^2 \omega^2}\right]^{1/2}}$$

For large ω V_{out} decreases

For $\omega \rightarrow 0$ $V_{out} \rightarrow V_{in}$

Power absorbed in AC circuit

(4)

$$V(t) = V_m \sin(\omega t)$$

$$I(t) = I_m \sin(\omega t + \phi)$$

$$I_m = \frac{V_m}{Z} \quad ; \quad \phi = \tan^{-1} \frac{X_C - X_L}{R}$$

$$P(t) = I(t) V(t)$$

$$= V_m I_m \sin \omega t \cdot \sin(\omega t + \phi)$$

$$= V_m I_m \left[\sin^2 \omega t \cos \phi + \sin \omega t \cos \omega t \sin \phi \right]$$

$$\langle P(t) \rangle = \frac{V_m \cdot I_m \cos \phi}{2} = \frac{I_m^2 Z}{2} \cos \phi$$

$$= \frac{V_m^2}{2Z} \cos \phi$$

$\cos \phi \rightarrow$ power factor

$$\langle P \rangle = \frac{I_m^2 Z}{2} \cos \phi$$

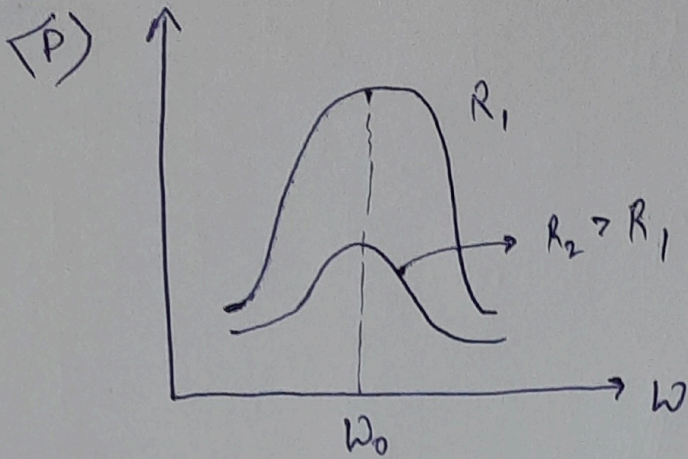
$$\tan \phi = \frac{X_C - X_L}{R}$$

$$\cos \phi = \frac{R}{Z}$$

$$\langle P \rangle = \frac{I_m^2 Z}{2} \cdot \frac{R}{Z} = \frac{I_m^2 R}{2} = I_{rms}^2 \cdot R$$

$\langle P \rangle$ is maximum
at $X_L = X_C$

$$= \frac{V_{rms}^2}{R^2 + (X_L - X_C)^2} \cdot R$$



$\langle P \rangle$ is maximum at $X_L = X_C$

$$\langle P \rangle = \frac{V_{rms}^2}{R}$$

1. For a purely resistive circuit

$$\cos \phi = \frac{R}{Z} \Rightarrow 1 ; \phi = 0$$

2. For purely capacitive or inductive circuit

$$\phi = \pm \frac{\pi}{2} ; \cos \phi = 0$$

Wattless circuits

3. L-C-R circuit

$$\tan \phi = \frac{X_C - X_L}{R} \Rightarrow \phi \neq 0$$

Power dissipation through resistance only

4. Circuit at ~~resonance~~ resonance $X_L = X_C$, $\phi = 0$

Maximum power dissipation (through resistance only)