

Q3) If  $ax^2 + 2hxy + by^2 = 1$ , then  $\frac{d^2y}{dx^2}$  is

Solution:

$$ax^2 + 2hxy + by^2 = 1$$

Differentiating both sides w.r.t  $x$ , we get:

$$2ax + 2hx \frac{dy}{dx} + 2hy + 2by \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = - \frac{ax + hy}{hx + by}$$

Again differentiating w.r.t  $x$  we get, we get

$$\Rightarrow \frac{d^2y}{dx^2} = - \left[ \frac{(hx + by) \left( a + h \frac{dy}{dx} \right) - (ax + hy) \left( h + b \frac{dy}{dx} \right)}{(hx + by)^2} \right]$$

$$= - \left[ \frac{y(ab - h^2) + \frac{dy}{dx}(h^2x - abx)}{(hx + by)^2} \right]$$

$$= \frac{h^2 - ab \left( y - x \frac{dy}{dx} \right)}{(hx + by)^2} = \frac{(h^2 - ab)}{(hx + by)^2} \left[ y + x \left( \frac{ax + by}{hx + by} \right) \right]$$

$$= \frac{h^2 - ab}{(hx + by)^2}$$