Q3) If $f(x) = \log_x (\log x)$, then $f'(x)$ at $x = e$ is _____

[IIT JEE 1985]

Solution :

$$f(x) = \log_x (\log x) = \frac{\log_e (\log_e x)}{\log_e x}$$

$$f'(x) = \frac{\dfrac{1}{\log_e x} \cdot \dfrac{1}{x} \log_e x - \dfrac{1}{x} \log_e (\log_e x)}{(\log_e x)^2}$$

$$= \frac{\dfrac{1}{x} \left[1 - \log_e (\log_e x) \right]}{(\log_e x)^2}$$

at $x = e$, we get $\Rightarrow$ $f'(e) = \dfrac{\dfrac{1}{e} \left[1 - \log_e (\log_e e) \right]}{(\log_e e)^2} = \dfrac{1}{e}$

Ans