Tuesday, March 1, 2022 12:16 PM
Q2) If
$$n = cosec \theta - sin\theta$$
 and $y = cosec \theta - sin^{n}\theta$, then show that
 $\left[(x^{2} + 4) (\frac{dy}{dx})^{2} = n^{2} (y^{2} + 4) \right]$ [IITJEE 1989]

Solutions As
$$x = (asec \theta - kin \theta)^2 + 4 = (cosec \theta + kin \theta)^2 - (1)$$

 $\Rightarrow x^2 + 4 = (cosec \theta - kin \theta)^2 + 4 = (cosec \theta + kin \theta)^2 - (2)$
 $y^2 + 4 = (cosec^{n} \theta - kin^{n} \theta)^2 + 4 = (cosec^{n} \theta + kin^{n} \theta)^2 - (2)$

Now,

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{n(ane^{n+\theta})(-consecced\theta) - nein^{n+\theta} e^{-\theta}}{-c\theta e^{-\theta} e^{-\theta}}$$

$$= \frac{n(consec^{n}\theta col\theta + lin^{n+\theta} col\theta)}{(consec\theta col\theta + col\theta)}$$

$$= n(consec^{n}\theta + lin^{n+\theta}) = \frac{n\sqrt{y^{2}+4}}{\sqrt{x^{2}+4}} (from 1 d 2)$$

Squaring Both kides we get,

$$\left[\left(\frac{1}{2} + 4 \right) \left(\frac{1}{2} \frac{1}{2} \right) = n^2 \left(\frac{1}{2} + 4 \right) \right] \frac{1}{2}$$