

5. Find the equation of the common tangent in 1st quadrant to the circle $x^2 + y^2 = 16$ and the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$. Also find the length of the intercept of the tangent between the coordinate axes. *(2005 - 4 Marks)*

Solution: -

5. Let the common tangent to circle $x^2 + y^2 = 16$ and ellipse $x^2/25 + y^2/4 = 1$ be

$$y = mx + \sqrt{25m^2 + 4} \quad \text{..(i)}$$

As it is tangent to circle $x^2 + y^2 = 16$, we should have

$$\frac{\sqrt{25m^2 + 4}}{\sqrt{m^2 + 1}} = 4$$

[Using : length of perpendicular from (0,0) to (1) = 4]

$$\Rightarrow 25m^2 + 4 = 16m^2 + 16 \Rightarrow 9m^2 = 12$$

$$\Rightarrow m = \frac{-2}{\sqrt{3}}$$

[Leaving + ve sign to consider tangent in I quadrant]

\therefore Equation of common tangent is

$$y = -\frac{2}{\sqrt{3}}x + \sqrt{25 \cdot \frac{4}{3} + 4} \Rightarrow y = -\frac{2}{\sqrt{3}}x + 4\sqrt{\frac{7}{3}}$$

This tangent meets the axes at $A(2\sqrt{7}, 0)$ and $B\left(0, 4\sqrt{\frac{7}{3}}\right)$

\therefore Length of intercepted portion of tangent between axes

$$= AB = \sqrt{(2\sqrt{7})^2 + \left(4\sqrt{\frac{7}{3}}\right)^2} = 14/\sqrt{3}$$