

Notes: Logarithms

Conversion between logarithmic and exponential form:

$$\log_a b = x \leftrightarrow b = a^x \text{ if } a > 0, a \neq 1, b > 0$$

Properties of log:

$$(i) \log(ab) = \log a + \log b, \text{ or in general: } \log\left(\prod_{i=1}^n a_i\right) = \sum_{i=1}^n \log a_i$$

$$(ii) \log \frac{a}{b} = \log a - \log b$$

$$(iii) \log a^n = n \log a$$

$$(iv) \log_b^n a = \frac{1}{n} \log_b a$$

$$(v) \log_b a = \frac{\log_c a}{\log_c b}, \text{ where } c > 0, c \neq 1$$

Limits related to log:

$$\lim_{x \rightarrow 0} \frac{\log_a(1+x)}{x} = \log_e a$$

$$\lim_{x \rightarrow \infty} \frac{\log x}{x} = 0$$

Derivative of log:

$$\frac{d}{dx} (\log_a x) = \frac{d}{dx} \left(\frac{\log_e x}{\log_e a} \right) = \frac{1}{x \log_e a}$$