1) $E(c) = c$

$$E(c) = c \cdot p_1 + c \cdot p_2 + \dots + c \cdot p_n$$
$$= c(p_1 + p_2 + \dots + p_n)$$
$$= c \cdot 1 \quad [\because p_1 + p_2 + \dots + p_n = 1]$$
$$= c$$

2) $E(ax+b) = aE(x)+b$

$$E(ax+b) = (ax_1+b)p_1 + (ax_2+b)p_2 + \dots + (ax_n+b)p$$
$$= a(x_1p_1 + x_2p_2 + \dots + x_np_n) + b(p_1+p_2+\dots+p_n)$$
$$= aE(x)+b$$

3) $var(x) = E(x-\mu)^2$
$$= E(x^2) - \{E(x)\}^2$$

$$Var(x) = E(x-\mu)^2$$
$$= E(x^2 - 2\mu x + \mu^2] \dots (1)$$
$$= E(x^2) - 2\mu E(x) + \mu^2$$
$$= E(x^2) - 2\mu^2 + \mu^2$$
$$= E(x^2) - \mu^2$$
$$= E(x^2) - \{E(x)\}^2$$

4) Standard deviation of a random variable X is defined as

$$s.d(X) = \sqrt{Var(X)}$$

5) Binomial distribution for repeated experiments:

If the probability of success of an event in one trial is 'p', and that of its failure is 'q', so that $p+q=1$, then the probability of ~~ex success~~ exactly r successes in n trials is

$$P(X=r) = {}^{n}C_{r}\, p^{r} q^{n-r}, \quad r = 0, 1, 2, \ldots, n$$

6) $$\sum_{k=0}^{n} p_k = \sum_{k=0}^{n} {}^{n}C_{k}\, p^{k} q^{n-k}$$

$$= \binom{n}{0} q^{n} + \binom{n}{1} p q^{n-1} + \binom{n}{2} p^{2} q^{n-2} + \ldots + \binom{n}{n} p^{n}$$

$$= (q+p)^{n} = 1^{n} = 1$$

7) $$\mu = E(X) = \sum_{k=0}^{n} k\, p_k = \sum_{k=0}^{n} k \binom{n}{k} p^{k} q^{n-k}$$

$$= \sum_{k=1}^{n} k \binom{n}{k} p^{k} q^{n-k}$$

$$= \sum_{k=1}^{n} k \cdot \frac{n!}{k!(n-k)!} p^k q^{n-k}$$

$$= \sum_{k=1}^{n} \frac{n! \, p^k q^{n-k}}{(k-1)!(n-1-(k-1))!}$$

$$= np \sum_{m=0}^{n-1} \frac{(n-1)! \, p^{k-1} q^{n-1-m}}{m!(n-1-m)!} \quad (\text{let } k-1 = m)$$

$$= np \sum_{m=0}^{n-1} \binom{n-1}{m} p^{m} q^{n-1-m}$$

$$= np(q+p)^{n-1} = np$$

8) $$E(x(x-1)) = \sum_{k=0}^{n} k(k-1) \binom{n}{k} p^k q^{n-k}$$

$$= \sum_{k=2}^{n} k(k-1) \binom{n}{k} p^k q^{n-k}$$

$$= \sum_{k=2}^{n} k(k-1) \frac{n! \, p^k q^{n-k}}{k!(n-k)!}$$

$$= \sum_{k=2}^{n} \frac{n! \, p^k q^{n-k}}{(k-2)!(n-k)!}$$

$$= n(n-1) \sum \frac{(n-2)! \, p^{k-2}}{(k-2)!(n-2-(k-2))!}$$

$$= n(n-1)p^2 \sum_{m=0}^{n-2} \frac{(n-2)!\,p^m q^{n-2-m}}{m!(n-2-m)!} \quad (\text{let } k-2=m)$$

$$= n(n-1)p^2(q+p)^{n-2}$$

$$= n(n-1)p^2$$

$$E(x^2) = E\{x(x-1)+x\}$$

$$= E(x(x-1)) + E(x)$$

$$E(x^2) = n(n-1)p^2 + np$$

$$Var(x) = E(x^2) - \{E(x)\}^2$$

$$= n(n-1)p^2 + np - n^2p^2$$

$$= np - np^2$$

$$= np(1-p)$$

$$= npq$$