Important form of MODULUS ⇒

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x \leq 0 \end{cases} \qquad x \in \mathbb{R}$$

1. if $a \in (0, \infty)$, then

(I) $|x| > a \Rightarrow x < -a$ or $x > a$

i.e. $x \in (-\infty, -a)$ or $x \in (a, \infty)$

$\Rightarrow x \in (-\infty, -a) \cup (a, \infty)$

(II) $|x| \geq a \Rightarrow x \leq -a$ or $x \geq a$

i.e. $\Rightarrow x \in (-\infty, -a] \cup [a, \infty)$

(III) $|x| < a \Rightarrow -a < x < a$

i.e. $x \in (-a, a)$

(IV) $|x| \leq a \Rightarrow -a \leq x \leq a$

i.e. $x \in [-a, a]$

2. LET $\underline{r}$ be positive real number and $\underline{a}$ be a fixed real number then

(I) $|x-a| < r \Rightarrow a-r < x < a+r$

i.e. $x \in (a-r, a+r)$

(II) $|x-a| \leq r \Rightarrow a-r \leq x \leq a+r$

i.e. $x \in [a-r, a+r]$

(III) $|x-a| > r \Rightarrow x < a-r$ or $x > a+r$

i.e. $x \in (-\infty, a-r) \cup (a+r, \infty)$

(IV) $|x-a| \geq r \Rightarrow x \leq a-r$ or $x \geq a+r$

i.e. $x \in [-\infty, a-r] \cup [a+r, \infty)$