

2. solve for x , $\frac{1}{|x|-3} \leq \frac{1}{2}$

Solⁿ \Rightarrow

$$\frac{1}{|x|-3} \leq \frac{1}{2}$$

$$\Rightarrow \frac{1}{|x|-3} - \frac{1}{2} \leq 0$$

$$\Rightarrow \frac{2 - (|x|-3)}{2(|x|-3)} \leq 0$$

$$\Rightarrow \frac{5 - |x|}{|x|-3} \leq 0$$

$$\Rightarrow \frac{|x|-5}{|x|-3} \geq 0 \quad (\text{multiplying both sides by } -1)$$

CASE-1 \Rightarrow $|x|-5 \geq 0$ and $|x|-3 > 0$.

$$\begin{aligned} &\Rightarrow |x| \geq 5 \text{ and } |x| > 3 \\ &\Rightarrow (x \leq -5 \text{ or } x \geq 5) \text{ and } (x < -3 \text{ or } x > 3) \\ &\Rightarrow x \in (-\infty, -5] \cup [5, \infty) \cap (-\infty, -3] \cup [3, \infty) \\ &\Rightarrow x \in (-\infty, -5] \cup [5, \infty) \end{aligned}$$

CASE-2 \Rightarrow $|x|-5 \leq 0$ and $|x|-3 < 0$

$$\Rightarrow |x| \leq 5 \text{ and } |x|-3 < 0$$

$$\Rightarrow (-5 \leq x \leq 5) \text{ and } (-3 < x < 3)$$

$$\Rightarrow x \in [-5, 5] \text{ and } x \in (-3, 3)$$

$$\Rightarrow x \in [-5, 5] \cap (-3, 3)$$

$$\Rightarrow x \in (-3, 3)$$

union of both cases gives

$$x \in (-\infty, -5] \cup [5, \infty) \cup (-3, 3)$$