

1. Solve for x , $\frac{|x-2|-1}{|x-2|-2} \leq 0$ How

Use of formula $\frac{a}{b} \leq 0$ ($b \neq 0$)

then a & b are of the opposite signs and consider two cases.

CASE-1 $\Rightarrow (|x-2|-1 \leq 0)$ and $(|x-2|-2 > 0)$
{since $|x-2|-2 \neq 0$ }

(i) $\Rightarrow (-1 \leq x-2 \leq 1)$ and $(x-2 < -2)$ or $(x-2 > 2)$

$\Rightarrow (1 \leq x \leq 3)$ and $(x < 0)$ or $(x > 4)$

$\Rightarrow x \in [1, 3] \cap (-\infty, 0) \cup (4, \infty)$

Since there are no common values

$\Rightarrow x \in \{\emptyset\}$

CASE-2 $\Rightarrow (|x-2|-1 > 0)$ and $(|x-2|-2 < 0)$

$\Rightarrow (|x-2| > 1)$ and $(|x-2| < 2)$

$\Rightarrow (x-2 \leq -1)$ or $(x-2 \geq 1)$ and $(-2 < x-2 < 2)$

$\Rightarrow (x \leq 1)$ or $(x \geq 3)$ and $(0 < x < 4)$

$\Rightarrow x \in (-\infty, 1] \cup [3, \infty) \cap (0, 4)$

$\Rightarrow x \in (0, 1] \cup [3, 4)$

Hence the overall solution set of the given inequality will be the union of the sets obtained in two cases.

Hence, $x \in (0, 1] \cup [3, 4)$