Chapter 1

# RELATIONS AND FUNCTIONS

## 1.1 Overview

### 1.1.1 *Relation*

A relation R from a non-empty set A to a non empty set B is a subset of the Cartesian product A × B. The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R. The set of all second elements in a relation R from a set A to a set B is called the range of the relation R. The whole set B is called the codomain of the relation R. Note that range is always a subset of codomain.

### 1.1.2 *Types of Relations*

A relation R in a set A is subset of A × A. Thus empty set $\phi$ and A × A are two extreme relations.

(i) A relation R in a set A is called empty relation, if no element of A is related to any element of A, i.e., $R = \phi \subset A \times A$.

(ii) A relation R in a set A is called universal relation, if each element of A is related to every element of A, i.e., $R = A \times A$.

(iii) A relation R in A is said to be reflexive if $a\text{R}a$ for all $a \in A$, R is symmetric if $a\text{R}b \Rightarrow b\text{R}a$, $\forall\ a, b \in A$ and it is said to be transitive if $a\text{R}b$ and $b\text{R}c \Rightarrow a\text{R}c$ $\forall\ a, b, c \in A$. Any relation which is reflexive, symmetric and transitive is called an equivalence relation.

**Note:** An important property of an equivalence relation is that it divides the set into pairwise disjoint subsets called equivalent classes whose collection is called a partition of the set. Note that the union of all equivalence classes gives the whole set.

### 1.1.3 *Types of Functions*

(i) A function $f : X \to Y$ is defined to be one-one (or injective), if the images of distinct elements of X under $f$ are distinct, i.e.,

$$x_1, x_2 \in X,\ f(x_1) = f(x_2) \Rightarrow x_1 = x_2.$$

(ii) A function $f : X \to Y$ is said to be onto (or surjective), if every element of Y is the image of some element of X under $f$, i.e., for every $y \in Y$ there exists an element $x \in X$ such that $f(x) = y$.