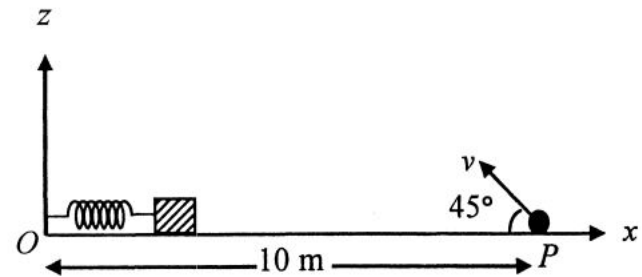
5. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad/s. Simultaneously at $t = 0$, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at $t = 1$ s, the value of v is (take $g = 10\ m/s^2$)



(A) $\sqrt{50}$ m/s (B) $\sqrt{51}$ m/s (C) $\sqrt{52}$ m/s (D) $\sqrt{53}$ m/s

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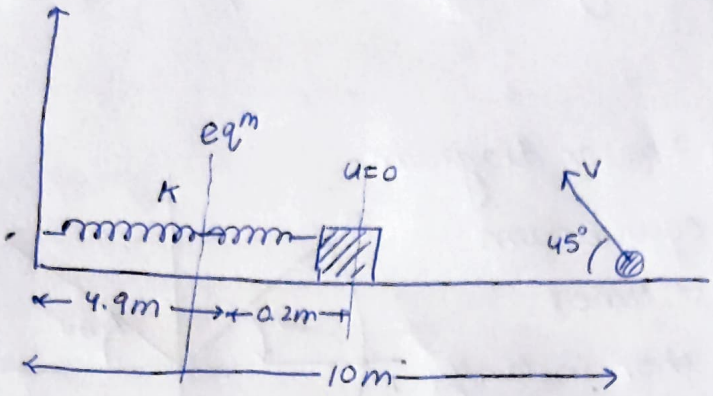
At $t=0$, both the masses were given motion.

So,



Given that both masses meet at $t=1s$,

so, we just need to find the position of mass which is doing SHM at $t=1s$.

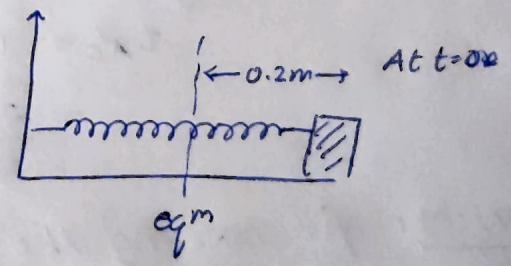
So, using the equation

$$x = A\cos\omega t$$

[$\cos\omega t$ is taken because at $t=0$, the block is at extreme position]

So, at $t=0$,

$x = 0.2\,m$ from eqm point



$0.2 = A\cos 0$

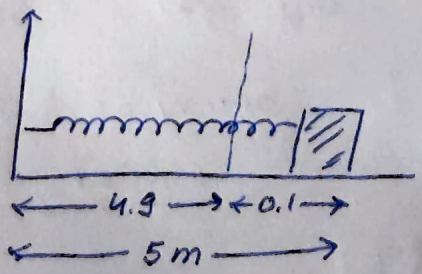
$\Rightarrow A = 0.2$

So, at $t=1$,

$$x = 0.2\cos(\omega) = 0.2\cos\left(\frac{\pi}{3}\right) \quad [\omega = \pi/3 \text{ GIVEN}]$$

$$= 0.1\,m$$

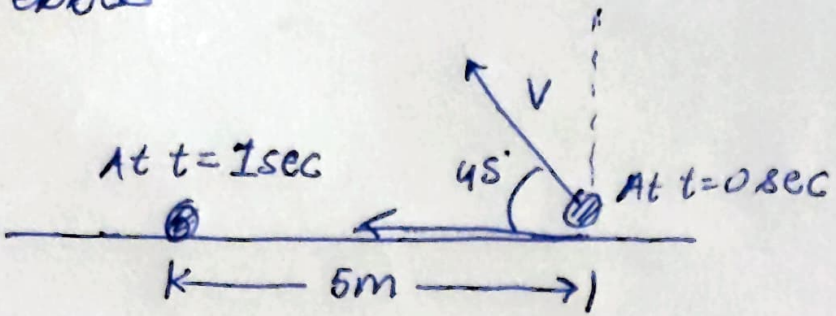
So, at $t=1s$.



so, the pebble needs to be at this position at $t=1s$.

i.e pebble needs to cover a distance horizontally $= 10-5 = 5m$.

Pebble



So, $V_x = V\cos 45 = \frac{V}{\sqrt{2}}$ & $x = 5m$ & $t = 1\,sec$

$\therefore \quad x = V_x * t \Rightarrow \quad 5 = \frac{V}{\sqrt{2}} \times 1 \Rightarrow$

$V = 5\sqrt{2}$

$= \sqrt{50}\ m/s$

OPTION (A) IS CORRECT ANSWER.