Table 6.2 Typical kinetic energies (*K*)

Object	Mass (kg)	Speed (m s^{-1})	K (J)
Car	2000	25	6.3×10^5
Running athlete	70	10	3.5×10^3
Bullet	5×10^{-2}	200	10^3
Stone dropped from 10 m	1	14	10^2
Rain drop at terminal speed	3.5×10^{-5}	9	1.4×10^{-3}
Air molecule	$\simeq10^{-26}$	500	$\simeq10^{-21}$

object can do by the virtue of its motion. This notion has been intuitively known for a long time. The kinetic energy of a fast flowing stream has been used to grind corn. Sailing ships employ the kinetic energy of the wind. Table 6.2 lists the kinetic energies for various objects.

Example 6.4 In a ballistics demonstration a police officer fires a bullet of mass 50.0 g with speed 200 m s^{-1} (see Table 6.2) on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet ?

Answer The initial kinetic energy of the bullet is $mv^2/2$ = 1000 J. It has a final kinetic energy of 0.1×1000 = 100 J. If v_f is the emergent speed of the bullet,

$$\frac{1}{2} m v_f^2 = 100 \text{ J}$$

$$v_f = \sqrt{\frac{2\times100 \text{ J}}{0.05 \text{ kg}}}$$

$$= 63.2 \text{ m s}^{-1}$$

The speed is reduced by approximately 68% (not 90%).

6.5 WORK DONE BY A VARIABLE FORCE

A constant force is rare. It is the variable force, which is more commonly encountered. Fig. 6.2 is a plot of a varying force in one dimension.

If the displacement Δx is small, we can take the force $F(x)$ as approximately constant and the work done is then

$$\Delta W = F(x)\,\Delta x$$

This is illustrated in Fig. 6.3(a). Adding successive rectangular areas in Fig. 6.3(a) we get the total work done as

$$W \cong \sum_{x_i}^{x_f} F(x)\Delta x \qquad (6.6)$$

where the summation is from the initial position x_i to the final position x_f.

If the displacements are allowed to approach zero, then the number of terms in the sum increases without limit, but the sum approaches a definite value equal to the area under the curve in Fig. 6.3(b). Then the work done is

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F(x)\Delta x$$

$$= \int_{x_i}^{x_f} F(x)\, dx \qquad (6.7)$$

where '*lim*' stands for the limit of the sum when Δx tends to zero. Thus, for a varying force the work done can be expressed as a definite integral of force over displacement (see also Appendix 3.1).

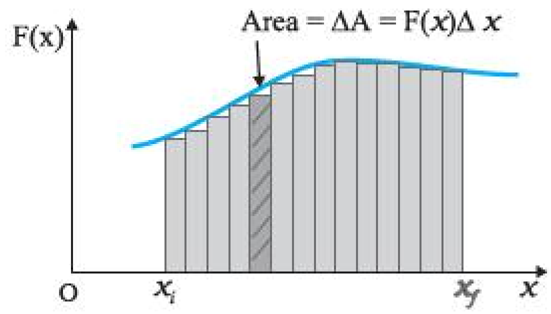
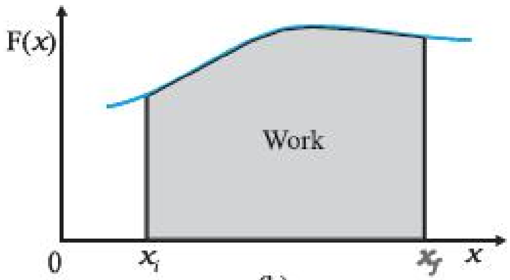


Fig. 6.3(a)



(b)

Fig. 6.3 *(a) The shaded rectangle represents the work done by the varying force F(x), over the small displacement Δx, ΔW = F(x) Δx. (b) adding the areas of all the rectangles we find that for Δx → 0, the area under the curve is exactly equal to the work done by F(x).*

> **Example 6.5** A woman pushes a trunk on a railway platform which has a rough surface. She applies a force of 100 N over a distance of 10 m. Thereafter, she gets progressively tired and her applied force reduces linearly with distance to 50 N. The total distance through which the trunk has been moved is 20 m. Plot the force applied by the woman and the frictional force, which is 50 N versus displacement. Calculate the work done by the two forces over 20 m.

Answer

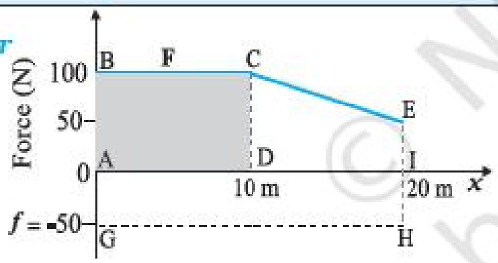


Fig. 6.4 *Plot of the force F applied by the woman and the opposing frictional force f versus displacement.*

The plot of the applied force is shown in Fig. 6.4. At x = 20 m, F = 50 N ($\neq 0$). We are given that the frictional force f is $|\mathbf{f}| = 50$ N. It opposes motion and acts in a direction opposite to **F**. It is therefore, shown on the negative side of the force axis.

The work done by the woman is

$W_F \rightarrow$ area of the rectangle ABCD + area of the trapezium CEID

$$W_F = 100 \times 10 + \frac{1}{2}(100 + 50) \times 10$$
$$= 1000 + 750$$
$$= 1750 \text{ J}$$

The work done by the frictional force is

$W_f \rightarrow$ area of the rectangle AGHI

$$W_f = (-50) \times 20$$
$$= -1000 \text{ J}$$

The area on the negative side of the force axis has a negative sign. ◀

6.6 THE WORK-ENERGY THEOREM FOR A VARIABLE FORCE

We are now familiar with the concepts of work and kinetic energy to prove the work-energy theorem for a variable force. We confine ourselves to one dimension. The time rate of change of kinetic energy is

$$\frac{\mathrm{d}K}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{2}m\,v^2\right)$$

$$= m\frac{\mathrm{d}v}{\mathrm{d}t}v$$

$$= F\,v \quad \text{(from Newton's Second Law)}$$

$$= F\frac{\mathrm{d}x}{\mathrm{d}t}$$

Thus

$$\mathrm{d}K = F\mathrm{d}x$$

Integrating from the initial position (x_i) to final position (x_f), we have

$$\int_{K_i}^{K_f} \mathrm{d}K = \int_{x_i}^{x_f} F\mathrm{d}x$$

where, K_i and K_f are the initial and final kinetic energies corresponding to x_i and x_f.

or $$K_f - K_i = \int_{x_i}^{x_f} F\mathrm{d}x \qquad (6.8a)$$

From Eq. (6.7), it follows that

$$K_f - K_i = W \qquad (6.8b)$$

Thus, the WE theorem is proved for a variable force.

While the WE theorem is useful in a variety of problems, it does not, in general, incorporate the complete dynamical information of Newton's second law. It is an integral form of Newton's second law. Newton's second law is a relation between acceleration and force at any instant of time. Work-energy theorem involves an integral over an interval of time. In this sense, the temporal (time) information contained in the statement of Newton's second law is 'integrated over' and is

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