5. Consider two identical springs each of spring constant k and negligible mass compared to the mass M as shown. Fig.1 shows one of them and Fig. 2 shows their series combination. The ratios of time period of oscillation of the two SHM is $T_b/T_a = \sqrt{x}$, where value of x is ______. (Round off to the nearest integer)

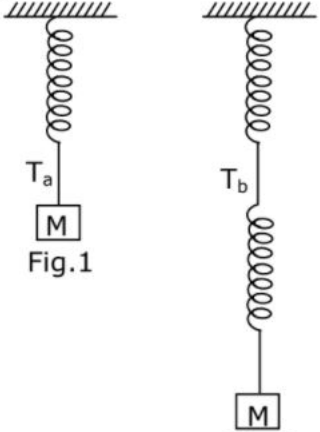
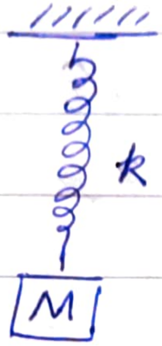


Fig.1 Fig.2

JEE MAINS 2021 (NUMERICAL TYPE)

SOLUTION :

For First case



from the lecture, we can see that

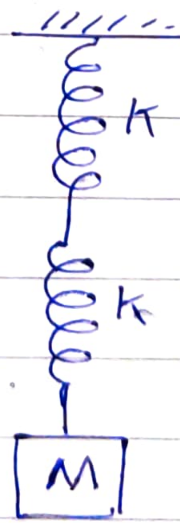
$$\omega^2 = \frac{k}{m}$$

i.e $\omega = \sqrt{\frac{k}{m}}$

so, now

$$\frac{2\pi}{T_a} = \omega \Rightarrow T_a = \frac{2\pi}{\omega} \Rightarrow T_a = 2\pi\sqrt{\frac{m}{k}}$$

For second case



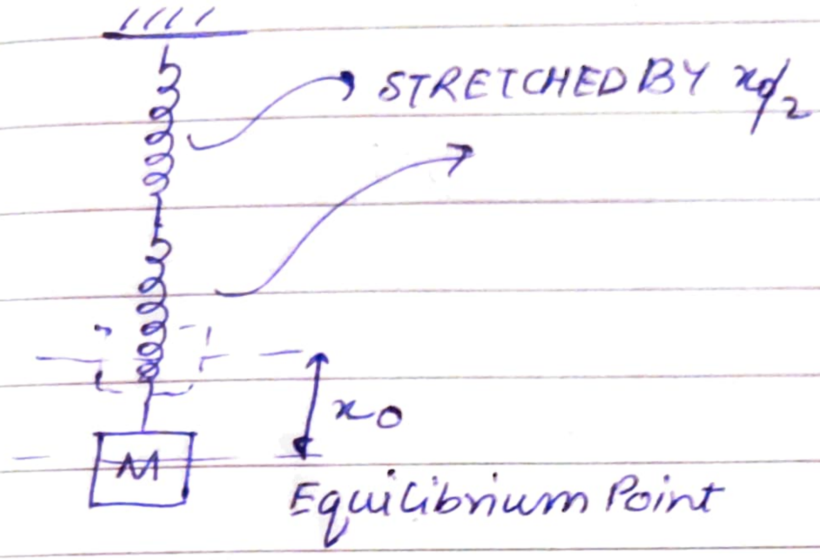
Now since the strings/springs are massless, so both the springs will experience same force 'Mg', and therefore will have same extension.

STEPS

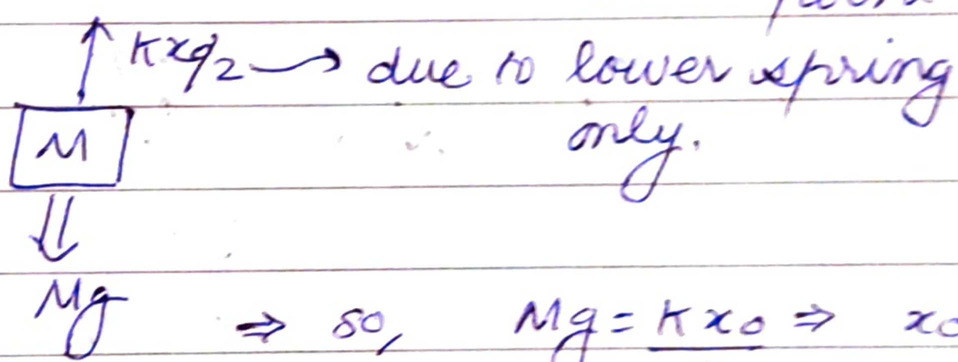
i> ANALYSE EQ^m POINT

ii> THEN EXTEND THE SPRING BY 'x'

iii> FIND RESTORING FORCE.

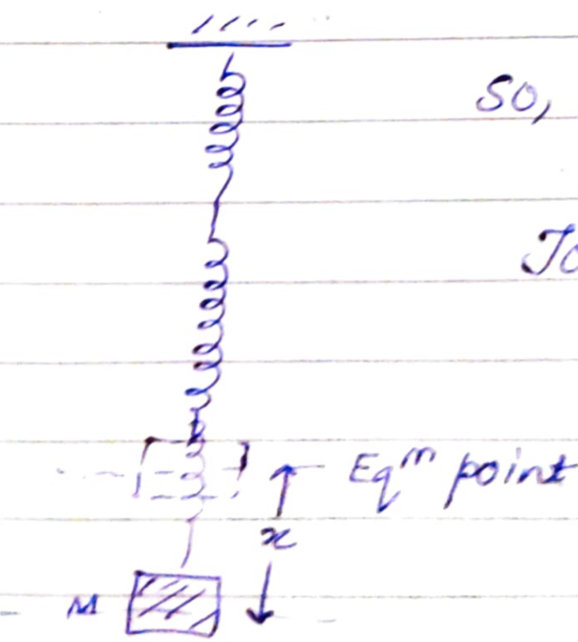


So, now analysing the equilibrium point.



$\Rightarrow$ so, $Mg = \frac{Kx_0}{2} \Rightarrow x_0 = \frac{2Mg}{K}$

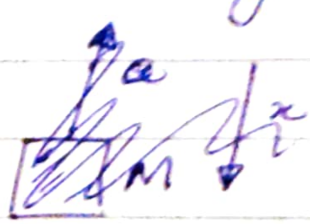
Now displacing the mass by 'x'



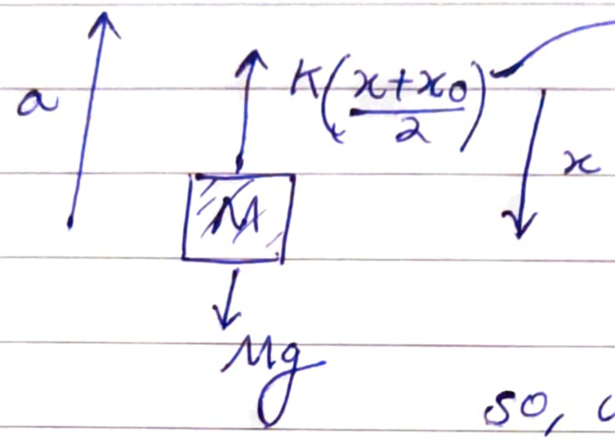
So, both string stretched by equal value,

Total stretch $= x + x_0$

each spring stretched by $\frac{x+x_0}{2}$



So, now drawing Free Body Diagram of 'M'



Due to lower spring only.

so, we have

$$ma = K\left(\frac{x+x_0}{2}\right) - mg$$

$$= \frac{Kx}{2} + \frac{Kx_0}{2} - mg$$

$$ma = \frac{Kx}{2} + \cancel{mg} - \cancel{mg}$$

AND here 'a' is in opposite direction of 'x' so, we can write

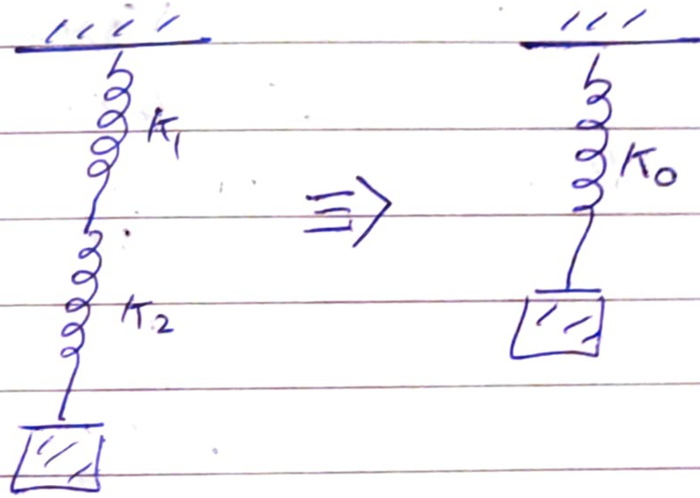
$$ma = -\frac{kx}{2}$$

$$a = \frac{-K x}{2m}, \text{ so } \omega = \sqrt{\frac{K}{2m}}$$

so, we get $T_b = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2m}{K}}$

$\frac{T_b}{T_a} = \sqrt{2}$, so, $\boxed{x = 2.}$

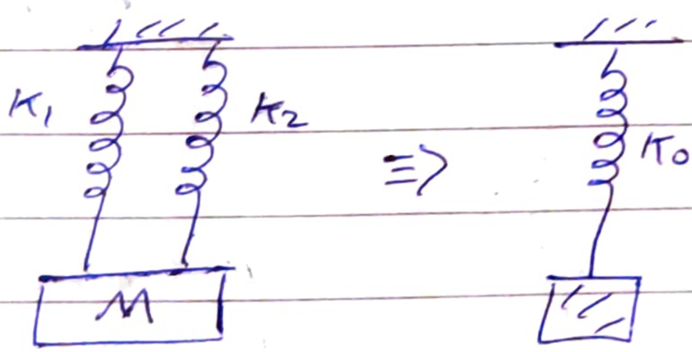
TIP: for combination of springs we have formula-



where

$$K_0 = \frac{K_1 K_2}{K_1 + K_2}$$

SERIES



where

$$K_0 = K_1 + K_2$$

PARALLEL.

So, in this case (in this question)

$$\omega_2 = \sqrt{\frac{K_{equivalent}}{m}}$$

$$K_{eq} = \frac{K_1 K_2}{K_1 + K_2} = \frac{K^2}{2K} = \frac{K}{2}$$

So, $\omega_2 = \sqrt{\frac{K}{2m}}$; $\boxed{T_0 = 2\pi\sqrt{\frac{2K}{m}}}$