LECTURE

NOTES

$$\cos^{-1} x = \tan^{-1}(?) \quad , \quad |x| \leq 1$$

Let $\cos^{-1}x = \theta \quad , \quad \theta \in [0, \pi]$

if $x \geq 0$, $\quad \theta \in \left[0, \frac{\pi}{2}\right] \subset \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\tan(\cos^{-1}x) = \tan\theta$$
$$= \frac{\sin\theta}{\cos\theta}$$
$$= \frac{\sqrt{1-\cos^2\theta}}{\cos\theta} \qquad (\because \sin^2\theta + \cos^2\theta = 1)$$

$$\tan(\theta) = \frac{\sqrt{1-x^2}}{x},$$

as $\theta \in$ Range set of $\tan^{-1}$

$$\theta = \cos^{-1}x = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right) \quad , \quad x \geq 0$$

if $-1 \leq x \leq 0$,

$$\theta = \cos^{-1}x \in \left(\frac{\pi}{2}, \pi\right]$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{1-\cos^2\theta}}{\cos\theta} = \frac{\sqrt{1-x^2}}{x}$$

$\theta \notin$ Range of $\tan^{-1}$

So, $\theta \notin \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

$$\tan\theta = \tan(\theta-\pi) \quad , \theta-\pi \in \left(-\frac{\pi}{2}, 0\right]$$

$\theta-\pi \in$ Range of $\tan^{-1}$

So, $\tan(\theta-\pi) = \dfrac{\sqrt{1-x^2}}{x}$

$$\theta-\pi = \tan^{-1}\left(\sqrt{\frac{1-x^2}{x^2}}\right)$$

$$\theta = \pi + \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$$

Finally,

$$\cos^{-1}x = \begin{cases} \tan^{-1}\left(\dfrac{\sqrt{1-x^2}}{x}\right) & , x \geq 0 \\ \pi + \tan^{-1}\left(\dfrac{\sqrt{1-x^2}}{x}\right) & , x < 0 \end{cases}$$

$$\tan^{-1}x = \cos^{-1}(?) \quad , x \in R$$

Let $x = \tan\theta$, $\tan^{-1}x = \theta$, $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\cos(\tan^{-1}x) = \cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{1+\tan^2\theta}}$$

$$\cos\theta = \cos(\tan^{-1}x) = \left(\frac{1}{\sqrt{1+x^2}}\right), \theta \in [0,\pi]$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) \quad , \quad \theta \in [0, \pi]$$

$\underline{x \geq 0 , \theta = \tan^{-1} x}$, $\theta \in \left[0, \frac{\pi}{2}\right)$

$\left[0, \frac{\pi}{2}\right) \subset \underbrace{[0, \pi]}_{\text{Range set of } \cos^{-1}}$

$\theta \in$ Range set of $\cos^{-1}$

$$\theta = \tan^{-1} x = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) \quad , \quad x \geq 0$$

$\underline{x < 0}$, $\theta = \tan^{-1} x$, $\theta \in \left(-\frac{\pi}{2}, 0\right)$

$\theta \notin [0, \pi]$, $\theta \notin$ Range set of $\cos^{-1}$

$\theta + \pi \in \left(\frac{\pi}{2}, \pi\right) \subset [0, \pi]$

$\theta + \pi \in$ Range set of $\cos^{-1}$

$$\cos(\theta + \pi) = -\cos\theta = -\frac{1}{\sqrt{1+x^2}}$$

$$\theta + \pi = \cos^{-1} \frac{-1}{\sqrt{1+x^2}} ,$$

$$\theta = -\pi + \cos^{-1}\left(\frac{-1}{\sqrt{1+x^2}}\right)$$

We can conclude,

$$\tan^{-1}x = \begin{cases} \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) & , \quad x \geq 0 \\ -\pi + \cos^{-1}\left(\frac{-1}{\sqrt{1+x^2}}\right) & , \quad x < 0 \end{cases}$$

To compute, $\sin^{-1}x + \sec^{-1}y$

$\downarrow$ $\downarrow$

convert it into, $\tan^{-1}(\ \ ) + \tan^{-1}(\ \ )$

and compute it.