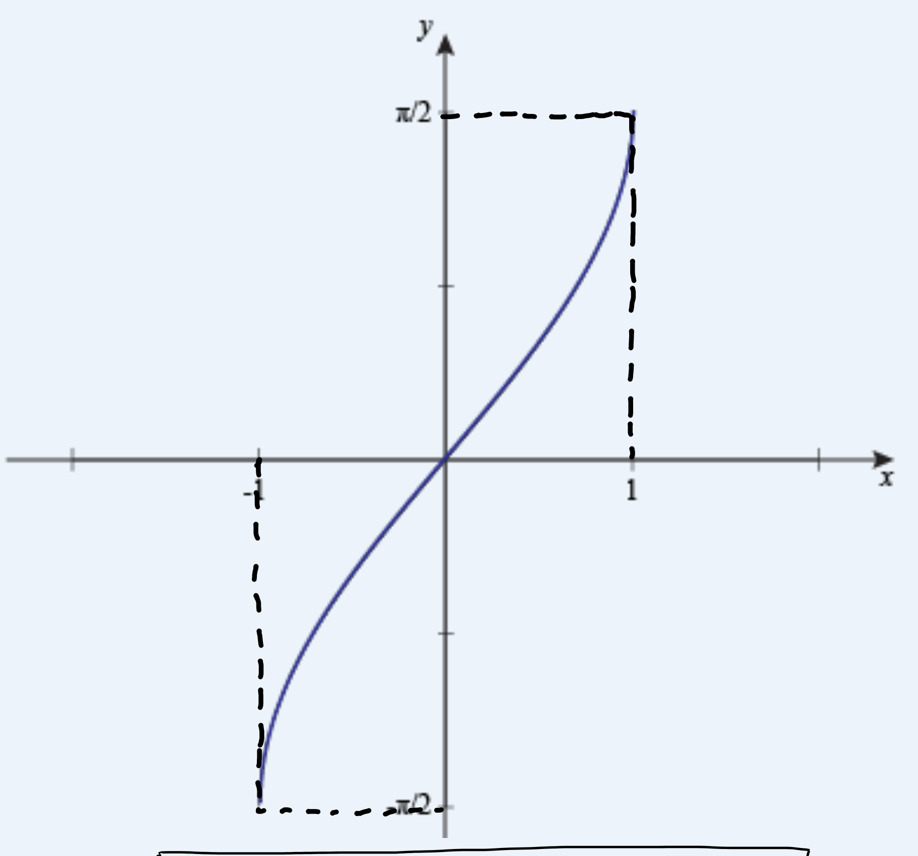
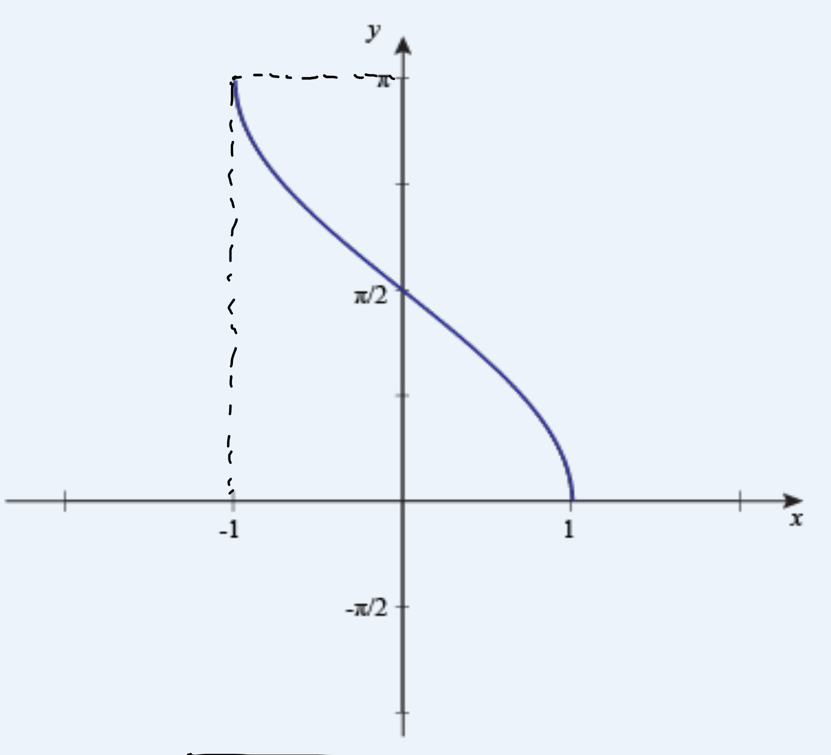
Inverse Trigonometric Functions important formulae

Friday, 11 February 2022   10:24 PM

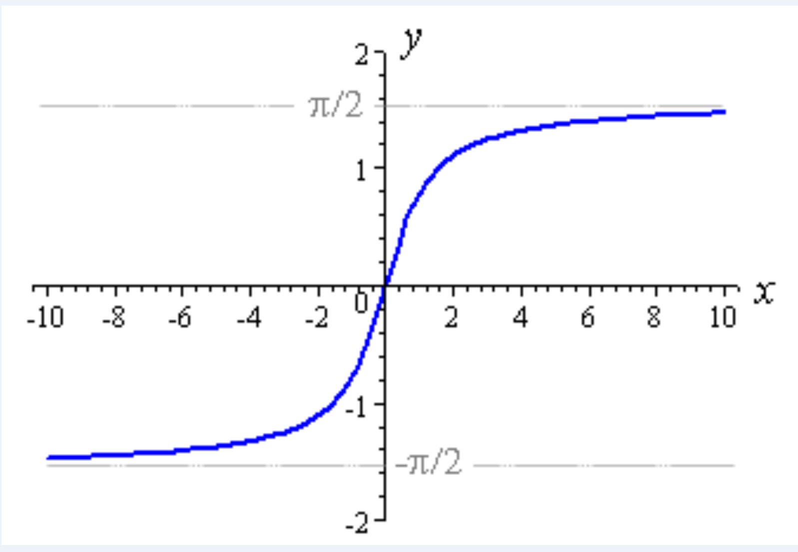
Graphs of Inverse Trigonometric Functions



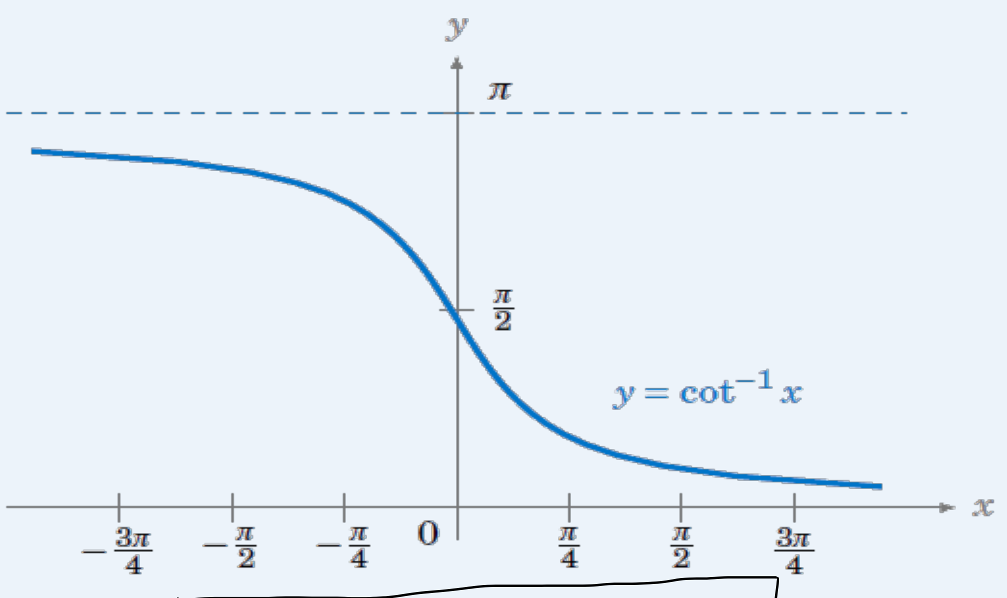
Graph of $y = \sin^{-1}(x)$



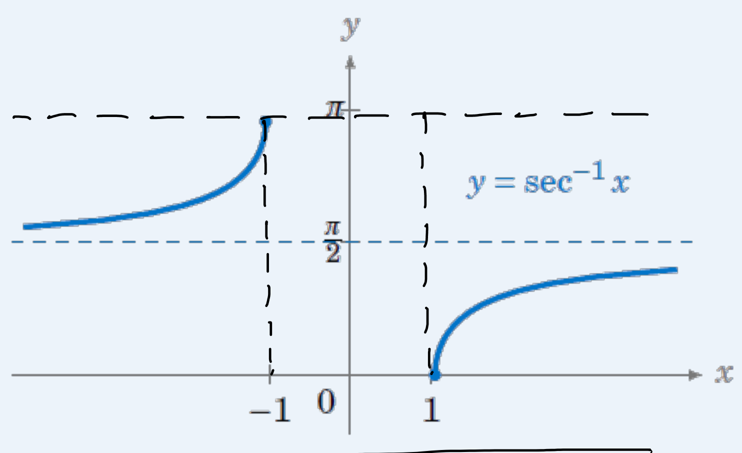
Graph of $y = \cos^{-1}(x)$



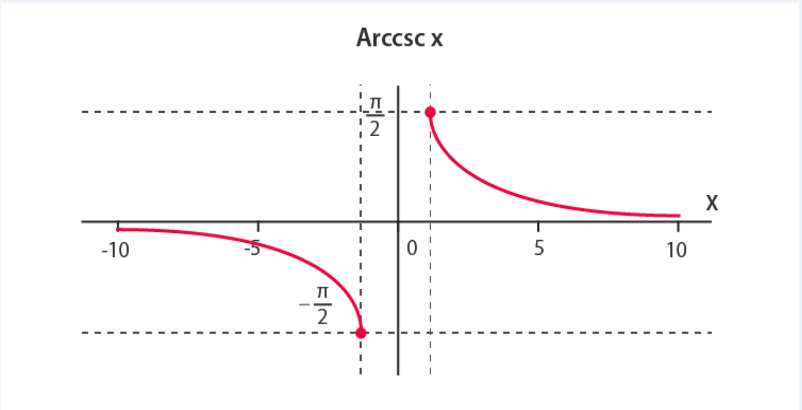
Graph of $y = \tan^{-1}(x)$



Graph of $y = \cot^{-1}(x)$



Graph of $y = \sec^{-1}(x)$



Graph of $y = \operatorname{cosec}^{-1}(x)$

Properties of Inverse Trigonometric Functions :

P-1.

1. $\sin(\sin^{-1}(x)) = x$ $|x| \le 1$ or $x \in [-1, 1]$
2. $\cos(\cos^{-1}(x)) = x$ $|x| \le 1$ or $x \in [-1, 1]$
3. $\tan(\tan^{-1}(x)) = x$ $x \in R$
4. $\cot(\cot^{-1}(x)) = x$ $x \in R$
5. $\sec(\sec^{-1}(x)) = x$ $|x| \geq 1$ or $x \in (-\infty, -1] \cup [1, \infty)$
6. $\operatorname{cosec}(\operatorname{cosec}^{-1}(x)) = x$ $|x| \geq 1$ or $x \in (-\infty, -1] \cup [1, \infty)$

P-2.

1. $\sin^{-1}(-x) = -\sin^{-1}(x)$ $x \in [-1, 1]$
2. $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ $x \in [-1, 1]$
3. $\tan^{-1}(-x) = -\tan^{-1}(x)$ $x \in R$
4. $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$ $x \in (-\infty, -1] \cup [1, \infty)$
5. $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}(x)$ $x \in (-\infty, -1] \cup [1, \infty)$
6. $\cot^{-1}(-x) = \pi - \cot^{-1}(x)$ $x \in R$

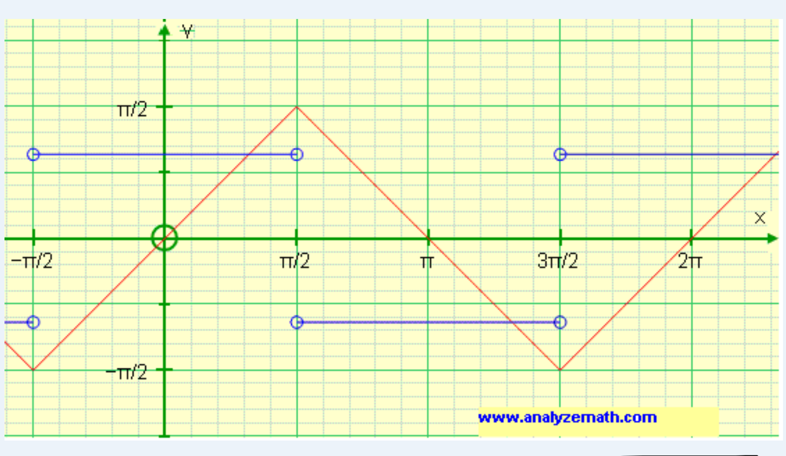
P-3.

1. $\sin^{-1}(x) = \operatorname{cosec}^{-1}(1/x)$; $x \in [-1, 1]$; $x \neq 0$
2. $\operatorname{cosec}^{-1}(x) = \sin^{-1}(1/x)$; $x \in (-\infty, -1] \cup [1, \infty)$
3. $\cos^{-1}(x) = \sec^{-1}(1/x)$; $x \in [-1, 1]$; $x \neq 0$
4. $\sec^{-1}(x) = \cos^{-1}(1/x)$; $x \in (-\infty, -1] \cup [1, \infty)$
5. $\tan^{-1}(x) = \begin{cases} \cot^{-1}(1/x) ; & x > 0 \\ -\pi + \cot^{-1}(1/x) ; & x < 0 \end{cases}$
6. $\cot^{-1}(x) = \begin{cases} \tan^{-1}(1/x) ; & x > 0 \\ \pi + \tan^{-1}(x) ; & x < 0 \end{cases}$

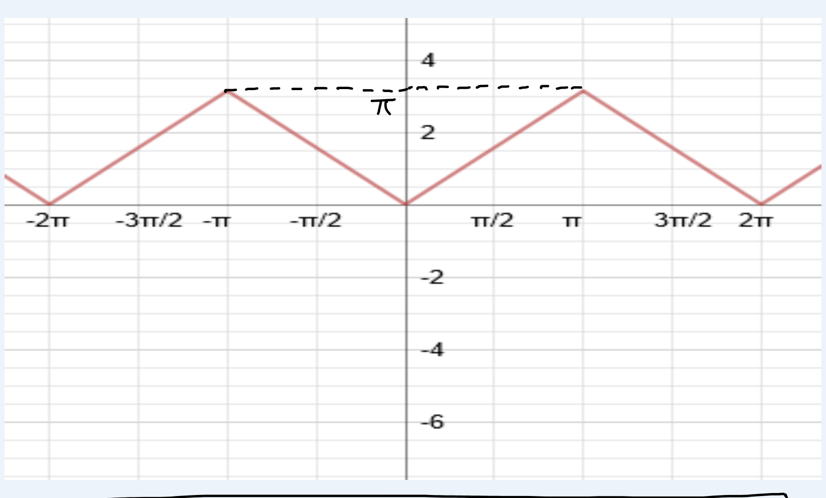
P-4 Vimp

1. $\sin^{-1}(x) + \cos^{-1}(x) = \pi/2$; $x \in [-1, 1]$
2. $\tan^{-1}(x) + \cot^{-1}(x) = \pi/2$; $x \in R$
3. $\sec^{-1}(x) + \operatorname{cosec}^{-1}(x) = \pi/2$; $x \in (-\infty, -1] \cup [1, \infty)$

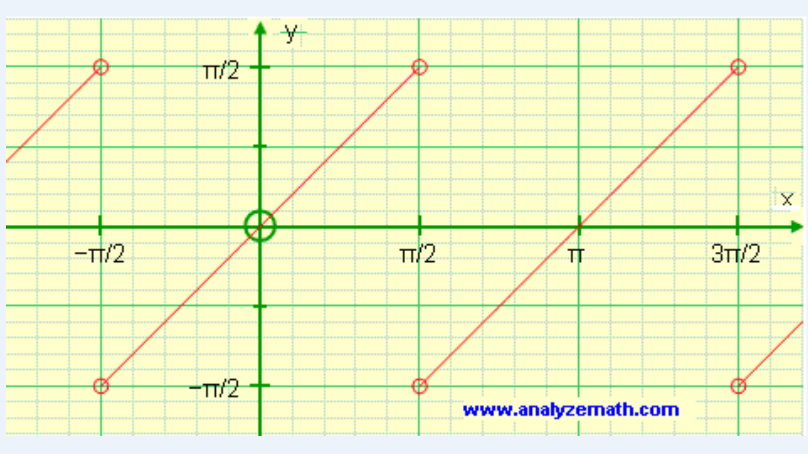
Graphs (Imp) :→



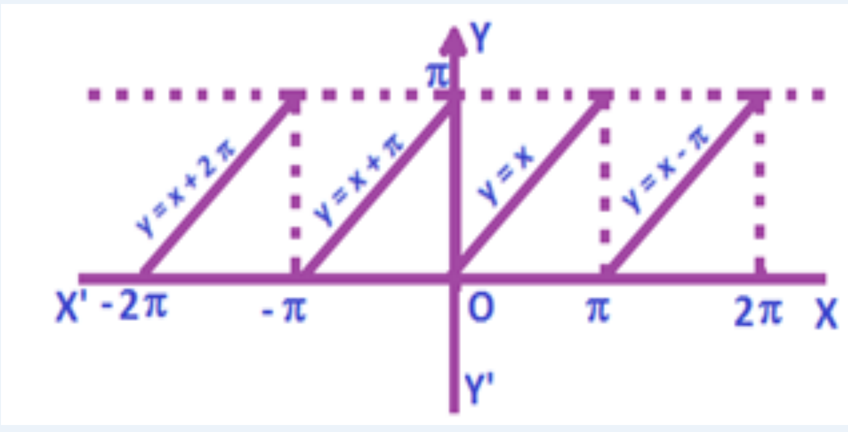
Graph of $y = \sin^{-1}(\sin x)$



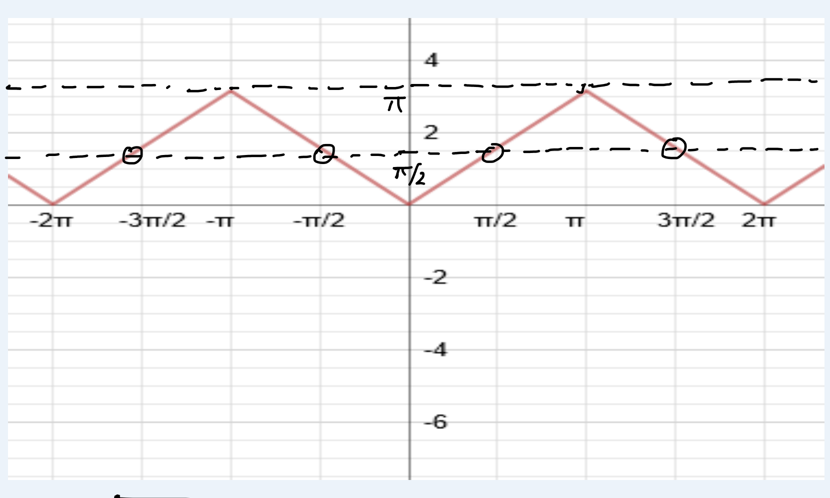
Graph of $y = \cos^{-1}(\cos x)$



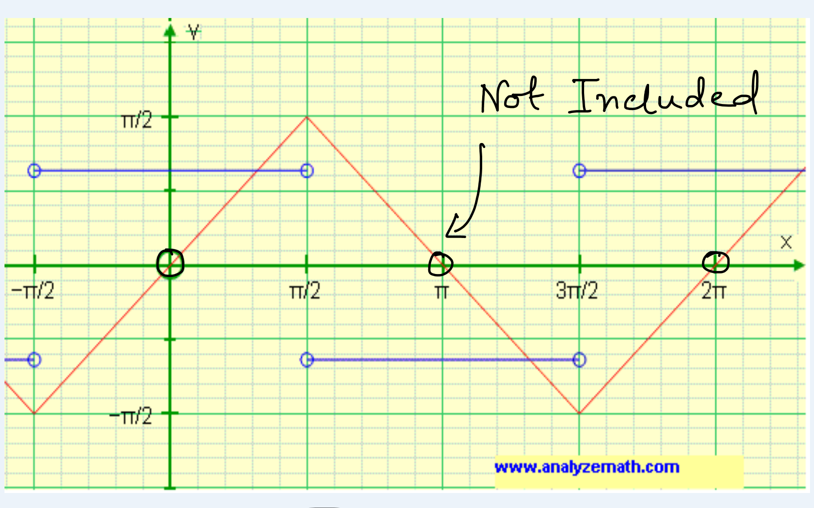
Graph of $y = \tan^{-1}(\tan x)$



Graph of $y = \cot^{-1}(\cot x)$



Graph of $y = \sec^{-1}(\sec x)$



Graph of $y = \operatorname{cosec}^{-1}(\operatorname{cosec} x)$

V. Imp Identities for JEE

1. $\tan^{-1}(x) - \tan^{-1}(y) = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$; $\boxed{x, y \geq 0}$
2. $\tan^{-1}(x) + \tan^{-1}(y) = \begin{cases} \tan^{-1}\left(\frac{x+y}{1-xy}\right) ; & x, y > 0 ,\ \ y < 1 \\ \pi + \tan^{-1}\left(\frac{x+y}{1-xy}\right) ; & x, y > 0 \ \ xy > 1 \end{cases}$
3. $\sin^{-1}(x) - \sin^{-1}(y) = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$ $\boxed{0 \le x, y \le 1}$
4. $\sin^{-1}(x) + \sin^{-1}(y) = \begin{cases} \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right) & 0 \le x, y \le 1,\ x^2+y^2 \le 1 \\ \pi - \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right) & 0 \le x, y \le 1,\ x^2+y^2 \geq 1 \end{cases}$
5. $\cos^{-1}(x) + \cos^{-1}(y) = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$ $0 \le x, y \le 1$
6. $\cos^{-1}(x) - \cos^{-1}(y) = \begin{cases} \cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right) & 0 \le x \le y \le 1 \\ -\cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right) & 0 \le y \le x \le 1 \end{cases}$