Determinants - Class XII

Related Questions with Solutions

Questions

Quetion: 01

The value of θ lying between 0 and $\frac{\pi}{2}$ and satisfying the equation $\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$ are

A. $\frac{5\pi}{24}, \frac{3\pi}{24}$
B. $\frac{7\pi}{24}, \frac{5\pi}{24}$
C. $\frac{7\pi}{24}, \frac{11\pi}{24}$
D. $\frac{\pi}{24}, \frac{11\pi}{24}$

Quetion: 02

The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ {}^{m}C_1 & {}^{m+1}C_1 & {}^{m+2}C_1 \\ {}^{m}C_2 & {}^{m+1}C_2 & {}^{m+2}C_2 \end{vmatrix}$ is equal to -

A. 1
B. -1
C. 0
D. None of these

Quetion: 03

If α, β, γ are the roots of $x^3 - 3x + 2 = 0,$ then the value of the determinant $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is equal to

A. -3
B. 2
C. 1
D. None of these

Quetion: 04

The value of the determinant $\begin{vmatrix} {}^5C_0 & {}^5C_3 & 14 \\ {}^5C_1 & {}^5C_4 & 1 \\ {}^5C_2 & {}^5C_5 & 1 \end{vmatrix}$ is

A. 0
B. $-(6!)$
C. 80
D. -576

Quetion: 05

The value of the determinant $\Delta = \begin{vmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{vmatrix}$ is equal to

A. 1
B. 0
C. 2
D. 3

Quetion: 06

If $f(x)=\begin{vmatrix}1 & x & x+1\\ 2x & x(x-1) & (x+1)x\\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1)\end{vmatrix}$, then $f(100)$ is equal to

A. 0
B. 1
C. 100
D. -100

Solutions

Solution: 01

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\Delta = \begin{vmatrix}1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta\\ -1 & 1 & 0\\ -1 & 0 & 1\end{vmatrix} = 0$$

$C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix}2+4\sin 4\theta & \cos^2\theta & 4\sin 4\theta\\ 0 & 1 & 0\\ 0 & 0 & 1\end{vmatrix} = 0$$

$\Rightarrow 4\sin 4\theta = -2$

$\Rightarrow \quad \sin 4\theta = -\frac{1}{2} = \sin\left(\frac{7\pi}{6}\right)$, sin $(11\pi/6)$

$\Rightarrow \quad \theta = \frac{7\pi}{24}, \frac{11\pi}{24}$

Solution: 02

$$\begin{vmatrix}1 & 1 & 1\\ {}^{m}C_1 & {}^{m+1}C_1 & {}^{m+2}C_1\\ {}^{m}C_2 & {}^{m+1}C_2 & {}^{m+2}C_2\end{vmatrix}$$

$$= \begin{vmatrix}1 & 1 & 1\\ {}^{m}C_1 & {}^{m+1}C_1 & {}^{m+1}C_0 + {}^{m+1}C_1\\ {}^{m}C_2 & {}^{m+1}C_2 & {}^{m+1}C_1 + {}^{m+1}C_2\end{vmatrix}$$

$$= \begin{vmatrix}1 & 1 & 0\\ {}^{m}C_1 & {}^{m+1}C_1 & {}^{m+1}C_0\\ {}^{m}C_2 & {}^{m+1}C_2 & {}^{m+1}C_1\end{vmatrix}$$ [Applying $C_3 \to C_3 - C_2$]

$$= \begin{vmatrix}1 & 1 & 0\\ {}^{m}C_1 & {}^{m}C_0 + {}^{m}C_1 & {}^{m+1}C_0\\ {}^{m}C_2 & {}^{m}C_1 + {}^{m}C_2 & {}^{m+1}C_1\end{vmatrix}$$

$$= \begin{vmatrix}1 & 0 & 0\\ {}^{m}C_1 & {}^{m}C_0 & {}^{m+1}C_0\\ {}^{m}C_2 & {}^{m}C_1 & {}^{m+1}C_1\end{vmatrix}$$ [Applying $C_2 \to C_2 - C_1$]

$={}^{m}C_0{}^{m+1}C_1 - {}^{m+1}C_0{}^{m}C_1$

= m + 1 – m = 1

Solution: 03

We have

$$\begin{vmatrix}\alpha & \beta & \gamma\\ \beta & \gamma & \alpha\\ \gamma & \alpha & \beta\end{vmatrix} = \begin{vmatrix}\alpha+\beta+\gamma & \beta & \gamma\\ \alpha+\beta+\gamma & \gamma & \alpha\\ \alpha+\beta+\gamma & \alpha & \beta\end{vmatrix}$$ $[C_1 \to C_1 + C_2 + C_3]$

= 0 $\quad [\because \alpha+\beta+\gamma = 0$ from the equation $x^3 - 3x + 2 = 0]$

Solution: 04

$$\begin{vmatrix} {}^5C_0 & {}^5C_3 & 14 \\ {}^5C_1 & {}^5C_4 & 1 \\ {}^5C_2 & {}^5C_5 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 10 & 14 \\ 5 & 5 & 1 \\ 10 & 1 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 16 & 16 & 16 \\ 5 & 5 & 1 \\ 10 & 1 & 1 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$= 16 \begin{vmatrix} 1 & 1 & 1 \\ 5 & 5 & 1 \\ 10 & 1 & 1 \end{vmatrix}$$

$$= 16 \begin{vmatrix} 0 & 0 & 1 \\ 0 & 4 & 1 \\ 9 & 0 & 1 \end{vmatrix} C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$$

= 16[0 – 36] = – 16 × 36

Solution: 05

$$\Delta = \begin{vmatrix} 1 & 4 & 9 & 16 \\ 4 & 9 & 16 & 25 \\ 9 & 16 & 25 & 36 \\ 16 & 25 & 36 & 49 \end{vmatrix} (R_3 \to R_3 - R_2, R_4 \to R_4 - R_1)$$

$$= \begin{vmatrix} 1 & 4 & 9 & 16 \\ 4 & 9 & 16 & 25 \\ 5 & 7 & 9 & 11 \\ 15 & 21 & 27 & 33 \end{vmatrix}$$

$$= 3 \begin{vmatrix} 1 & 4 & 9 & 16 \\ 4 & 9 & 16 & 25 \\ 5 & 7 & 9 & 11 \\ 5 & 7 & 9 & 11 \end{vmatrix} = 0 \quad (R_4 \to R_4 - R_3)$$

Solution: 06

We have

$$f(x) = x(x+1)(x-1) \begin{vmatrix} 1 & 1 & 1 \\ 2x & x-1 & x \\ 3x & x-2 & x \end{vmatrix}$$

$$= x(x+1)(x-1) \begin{vmatrix} 1 & 1 & 1 \\ 2x & x-1 & x \\ 3x & x-2 & x \end{vmatrix} \qquad [C_1 \to C_1 - C_3 \text{ and}$$

$C_2 \to C_2 - C_3] = 0$

Hence, $f(100) = 0$

Correct Options

Answer:01
Correct Options: C
Answer:02
Correct Options: A
Answer:03
Correct Options: D
Answer:04
Correct Options: D
Answer:05
Correct Options: B
Answer:06
Correct Options: A