Chapter 13 : Probability

13.3 Exercise

(*) Short Answer (S.A.).

1) Given that, $P(1) = P(2) = 0.2$, $P(4) = 0.3$.

$P(3) = P(5) = P(6) = 0.1$.

A: Same number each time.

Outcomes are $\{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ (6,6)\}$.

$\therefore P(A) = P(1,1) + P(2,2) + P(3,3) + P(4,4) + P(5,5) + P(6,6)$

$= P(1)(P(1)) + P(2)P(2) + P(3)(P(3)) + P(4)P(4) + P(5)P(5) + P(6)P(6)$.

$= (0.2)(0.2) + (0.2)(0.2) + (0.1)(0.1) + (0.3)(0.3) + (0.1)(0.1) + (0.1)(0.1)$

$= 0.2$

B: a Total score 10 or more.

Outcomes are $\{(4,6)\ (5,5)\ (5,6)\ (6,4)\ (6,5)\ (6,6)\}$.

$P(B) = P(4,6) + P(5,5) + P(5,6) + P(6,4) + P(6,5) + P(6,6)$.

$= P(4)P(6) + P(5)P(5) + P(5)P(6) + P(6)P(4) + P(6)P(5) + P(6)P(6)$.

$= (0.3)(0.1) + (0.1)(0.1) + (0.1)(0.1) + (0.1)(0.3) + (0.1)(0.1) + (0.1)(0.1)$

$= 0.1$.

Now, $A \cap B = \{(5,5)(6,6)\}$

$P(A \cap B) = P(5,5) + P(6,6) = P(5)P(5) + P(6)P(6)$

$= (0.1)(0.1) + (0.1)(0.1) = 0.02$.

$\therefore P(A \cap B) = 0.02.$

$P(A) \cdot P(B) = (0.2)(0.1) = 0.02.$

Hence, $P(A \cap B) = P(A) \cdot P(B)$

Therefore A and B are independent events.

(2)

We have, $A = \{(1,1)(2,2)(3,3)(4,4)(5,5)(6,6)\}.$

$\therefore n(A) = 6$ and $n(S) = 6 \times 6 = 36.$

[S is the sample space]

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}.$$

$B = \{(4,6)(6,4)(5,5)(6,5)(5,6)(6,6)\}.$

$\therefore n(B) = 6$ and $n(S) = 36.$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}.$$

$\Rightarrow A \cap B = \{(5,5)(6,6)\}.$

(*) $P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

$$P(A) \cdot P(B) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

$P(A \cap B) \neq P(A) \cdot P(B)$

Hence, A and B are not independent Events.

(4)

Bag contains 5 Red Balls and 3 Black Balls;

As said, First Ball / marble is red.

$$P(R_1) = \frac{5}{8}$$

Now, without replacement, 7 Marbles left.

Consider the following happenings.

E_1 = Second marble is Black, Third marble is Red.

E_2 = Second marble is Black, Third marble is Black

E_3 = Second marble is Red, Third marble is Black

* Note that other cases are ignored of the fact that atleast one marble should be black.

R_i = Drawing red ball in i^{th} draw.

B_i = Drawing Black ball in i^{th} draw.

$$\therefore P(E_1) = P(R_1) \cdot P(B_1/R_1) \cdot P(R_2/R_1B_1)$$

$$= \frac{5}{8} \cdot \frac{3}{7} \cdot \frac{4}{6} = \frac{5}{28}$$

$$P(E_2) = P(R_1) \cdot P(B_1/R_1) \cdot P(B_2/R_1B_1)$$

$$= \frac{5}{8} \cdot \frac{3}{7} \cdot \frac{2}{6} = \frac{5}{56}.$$

$$P(E_3) = P(R_1) \cdot P(R_2/R_1) \cdot P(B_1/R_1R_2)$$

$$= \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} = \frac{5}{28}.$$

$$P(E) = P(E_1) + P(E_2) + P(E_3)$$

→ Event E is the required Event

$$P(E) = \frac{5}{28} + \frac{5}{56} + \frac{5}{28} = \frac{25}{56}$$

(5)

Sample space (Ω) $= \begin{bmatrix} (1,1) & (1,2) & \cdots & (1,6) \\ (2,1) & (2,2) & \cdots & (2,6) \\ \vdots & & & \\ (6,1) & (6,2) & \cdots & (6,6) \end{bmatrix}$

$= 6 \times 6 = 36.$

E : A total of 4.

Outcomes are $\{(1,3)\ (2,2)\ (3,1)\}$.

$$P(E) = \frac{3}{36} = \frac{1}{12}.$$

F : A total of 9 or more.

Outcomes are $\{(3,6)\ (4,5)\ (4,6)\ (5,4)\ (5,5)\ (5,6)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$.

$$P(F) = \frac{10}{36} = \frac{5}{18}.$$

G: A total divisible by 5.

Outcomes are $\{(1,4)\ (2,3)\ (3,2)\ (4,1)\ (4,6)\ (5,5)\ (6,4)\}$.

$$P(G) = \frac{7}{36}.$$

* $E \cap F = \phi$ and $E \cap G = \phi$.

$$F \cap G = \{(4,6)\ (5,5)\ (6,4)\}.$$

$$P(F \cap G) = \frac{3}{36} = \frac{1}{12}.$$

$$P(F) \cdot P(G) = \frac{5}{18} \cdot \frac{7}{36} = \frac{35}{648}.$$

$$\therefore P(F \cap G) \neq P(F) \cdot P(G)$$

Hence There is <u>no pair</u> which is independent.