

Tips and Tricks:**Tip-1****Inverse of a matrix using determinants:**

The product of a matrix A and its adjoint is equal to unit matrix multiplied by the determinant A.

Let A be a square matrix, then (Adjoint A). A = A. (Adjoint A) = |A| . I

We know that, $A \cdot (\text{Adj } A) = |A|I$ or $\frac{A \cdot (\text{Adj } A)}{|A|} = I$ (Provided |A| \neq 0)

And $A \cdot A^{-1} = I$; $A^{-1} = \frac{1}{|A|} (\text{Adj } A)$

Illustration 2: If the product of a matrix A and $\begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$ is the matrix $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$,

then A^{-1} is given by:

$$(a) \begin{bmatrix} 0 & -1 \\ 2 & -4 \end{bmatrix} \quad (b) \begin{bmatrix} 0 & -1 \\ -2 & -4 \end{bmatrix} \quad (c) \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}$$

(d) None of these

This is an example of above trick.

Solution:

(a) We know if $AB = C$, then $B^{-1}A^{-1} = C^{-1} \Rightarrow A^{-1} = BC^{-1}$ by using this formula we will get value of A^{-1} in the above problem.

$$\text{Here, } A \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}$$

Tip-2**Determinant of an Adjoint matrix**

$\det(AB) = \det(A)\det(B)$ \longrightarrow **This is a useful property of det. worth remembering.**

$$\det(\text{Adj}(A)) = \det(A)^{(N-1)}$$

A is N x N matrix

Tip-3

Theorem: Inverse of A exists If and only if $\det(A)$ is non zero.