1 JEE Main 2021 (Online) 27th August Morning Shift

MCQ (Single Correct Answer)

If the matrix $A = \begin{pmatrix} 0 & 2 \\ K & -1 \end{pmatrix}$ satisfies $A(A^3 + 3I) = 2I$, then the value of K is :

A $\frac{1}{2}$

B $-\frac{1}{2}$

C -1

D 1

Explanation

Given matrix $A = \begin{bmatrix} 0 & 2 \\ k & -1 \end{bmatrix}$

$A^4 + 3IA = 2I$

$\Rightarrow A^4 = 2I - 3A$

Also characteristic equation of A is $|A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 0-\lambda & 2 \\ k & -1-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda + \lambda^2 - 2k = 0$

$\Rightarrow A + A^2 = 2K.I$

$\Rightarrow A^2 = 2KI - A$

$\Rightarrow A^4 = 4K^2I + A^2 - 4AK$

Put $A^2 = 2KI - A$

and $A^4 = 2I - 3A$

$2I - 3A = 4K^2I + 2KI - A - 4AK$

$\Rightarrow I(2 - 2K - 4K^2) = A(2 - 4K)$

$\Rightarrow -2I(2K^2 + K - 1) = 2A(1 - 2K)$

$\Rightarrow -2I(2K - 1)(K + 1) = 2A(1 - 2K)$

$\Rightarrow (2K - 1)(2A) - 2I(2K - 1)(K + 1) = 0$

$\Rightarrow (2K - 1)[2A - 2I(K + 1)] = 0$

$\Rightarrow K = \frac{1}{2}$

JEE Main 2021 (Online) 17th March Morning Shift

Numerical

If $A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$, then the value of det($A^4$) + det($A^{10}$ – (Adj(2A))^{10}) is equal to

______________.

Answer

Correct Answer is 16

Explanation

$A = \begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$

$|A| = -2 \Rightarrow |A|^4 = 16$

$A^2 = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 8 & 9 \\ 0 & -1 \end{bmatrix}$

$\therefore A^{10} = \begin{bmatrix} 2^{10} & 2^{10} - 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1024 & 1023 \\ 0 & 1 \end{bmatrix}$

$2A = \begin{bmatrix} 4 & 6 \\ 0 & -2 \end{bmatrix}$

$adj(2A) = \begin{bmatrix} -2 & -6 \\ 0 & 4 \end{bmatrix}$

$adj(2A) = -2\begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}$

$(adj(2A))^{10} = 2^{10}\begin{bmatrix} 1 & 3 \\ 0 & -2 \end{bmatrix}^{10}$

$= 2^{10}\begin{bmatrix} 1 & -(2^{10} - 1) \\ 0 & 2^{10} \end{bmatrix}$

$= 2^{10}\begin{bmatrix} 1 & -1023 \\ 0 & 1024 \end{bmatrix}$

$A^{10} - (adj(2A))^{10} = \begin{bmatrix} 0 & 2^{11} \times 1023 \\ 0 & 1 - (1024)^2 \end{bmatrix}$

$|A^{10} - adj(2A)^{10}| = 0$

$\therefore$ det(A^4) + det(A^{10} – (Adj(2A))^{10})

= 16 + 0 = 16

1 JEE Main 2021 (Online) 20th July Evening Shift

Numerical

Let $A = \{a_{ij}\}$ be a 3 $\times$ 3 matrix,

where $a_{ij} = \begin{cases} (-1)^{j-i} & if \quad i < j, \\ 2 & if \quad i = j, \\ (-1)^{i+j} & if \quad i > j \end{cases}$

then $\det(3Adj(2A^{-1}))$ is equal to ___________.

Answer

Correct Answer is **108**

Explanation

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$|A| = 4$$

$$\det(3adj(2A^{-1}))$$

$$= 3^3|adj(2a^{-1})|$$

$$= 3^2|2A^{-1}|^2$$

$$= 3^3.2^2|A^{-1}|^2 = 3^3.2^2.\frac{1}{|A|^2} = 3^2.2^2.\frac{1}{4^2} = 108$$

3 JEE Main 2021 (Online) 26th August Evening Shift

Numerical

Let A be a 3×3 real matrix. If $\det(2Adj(2\ Adj(Adj(2A)))) = 2^{41}$, then the value of $\det(A^2)$ equal __________.

Answer

Correct Answer is 4

Explanation

$adj\ (2A) = 2^2\ adjA$

$\Rightarrow adj(adj\ (2A)) = adj(4\ adjA) = 16\ adj\ (adj\ A)$

$= 16\ |\ A\ |\ A$

$\Rightarrow adj\ (32\ |\ A\ |\ A) = (32\ |\ A\ |)^2\ adj\ A$

$12(32|\ A\ |)^2\ |adj\ A\ | = 2^3\ (32\ |\ A\ |)^6\ |\ adj\ A\ |$

$2^3\ .\ 2^{30}\ |\ A\ |^6\ .\ |\ A\ |^2 = 2^{41}$

$|\ A\ |^8 = 2^8 \Rightarrow |\ A\ | = \pm 2$

$|\ A\ |^2 = |\ A\ |^2 = 4$

1 JEE Main 2021 (Online) 1st September Evening Shift

MCQ (Single Correct Answer)

Let $J_{n,m} = \int\limits_0^{\frac{1}{2}} \frac{x^n}{x^m - 1} dx$, $\forall$ n > m and n, m $\in$ N. Consider a matrix $A = [a_{ij}]_{3 \times 3}$ where $a_{ij} = \begin{cases} j_{6+i,3} - j_{i+3,3}, & i \le j \\ 0, & i > j \end{cases}$. Then $\left|adj A^{-1}\right|$ is :

A. $(15)^2 \times 2^{42}$

B. $(15)^2 \times 2^{34}$

C. $(105)^2 \times 2^{38}$

D. $(105)^2 \times 2^{36}$

Explanation

A = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

$J_{6+1,3} - J_{i+3,3}; i \le j$

$= \int_0^{\frac{1}{2}} \frac{x^{6+i}}{x^3 - 1} - \int_0^{\frac{1}{2}} \frac{x^{i+3}}{x^3 - 1}$

$= \int_0^{1/2} \frac{x^{i+3}(x^3 - 1)}{x^3 - 1}$

$= \frac{x^{3+i+1}}{3+i+1} = \left(\frac{x^{4+i}}{4+i}\right)_0^{1/2}$

$\therefore a_{ij} = j_{6+i,3} - j_{i+3,3} = \frac{\left(\frac{1}{2}\right)^{4+i}}{4+i}$

$a_{11} = \frac{\left(\frac{1}{2}\right)^5}{5} = \frac{1}{5.2^5}$

$a_{12} = \frac{1}{5.2^5}$

$a_{13} = \frac{1}{5.2^5}$

$a_{22} = \frac{1}{6.2^6}$

$a_{23} = \frac{1}{6.2^6}$

$a_{33} = \frac{1}{7.2^7}$

$A = \begin{bmatrix} \frac{1}{5.2^5} & \frac{1}{5.2^5} & \frac{1}{5.2^5} \\ 0 & \frac{1}{6.2^6} & \frac{1}{6.2^6} \\ 0 & 0 & \frac{1}{7.2^7} \end{bmatrix}$

$|A| = \frac{1}{5.2^5}\left[\frac{1}{6.2^6} \times \frac{1}{7.2^7}\right]$

$|A| = \frac{1}{210.2^{18}}$

$\left|adj A^{-1}\right| = \left|A^{-1}\right|^{n-1} = \left|A^{-1}\right|^2 = \frac{1}{(|A|)^2}$

$= (210.2^{18})^2$

= $(105)^2 \times 2^{38}$