Lecture 1 Notes

LINEAR Programming Problem

Linear Programming also called linear optimization is a method to use to achieve the best outcomes (Such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationship.

It is a special case of mathematical optimization.

Let us take some examples from real world -

(i) In a military operation.
(ii) In an industry
(iii) Salaried class.

LPP.

(i) Constraints are represented as linear equation / Linear inequation (one, two or more than two variables)

(ii) Plan of action (programming)

(i) & (ii) is Linear Programmy.

In all the above cases -

(i) Constraints are represented by linear equations/inequations (in one, two or more variables)

(ii) a particular plan of action from several alternatives is to be chosen (Programming)

(i) & (ii) steps together is called linear programming. So, Linear Programming is a method for determining optimum values of a linear function subject to constraints as linear equations or inequations.

* Some Definitions:—

1. Objective function:
2. Decision Variables:
3. Constraints:
4. Optimization Problem
5. Feasible Region
6. Feasible Solution

Objective function:

Linear function $Z = ax + by$ where a, b are constant is called objective function which have to be maximize or minimize.

2. Decision variable:

x & y are called decision variable.

$x \geq 0$, $y \geq 0$

(x & y have non-negative restriction).

3. Constraints: Condition/hurdles

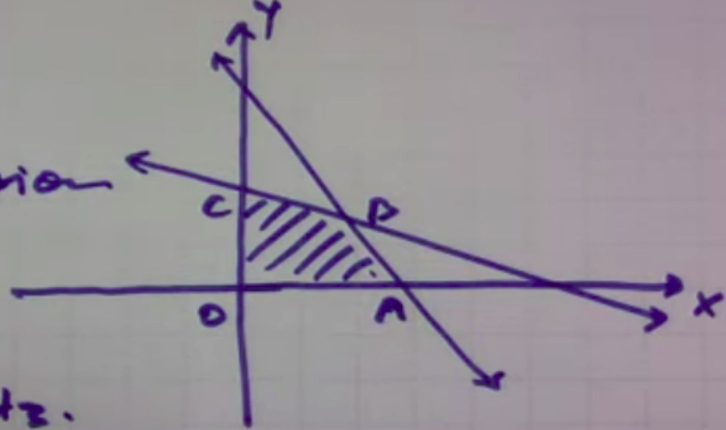
Linear equation / Linear inequation & ~~decision~~ condition on disision variables.

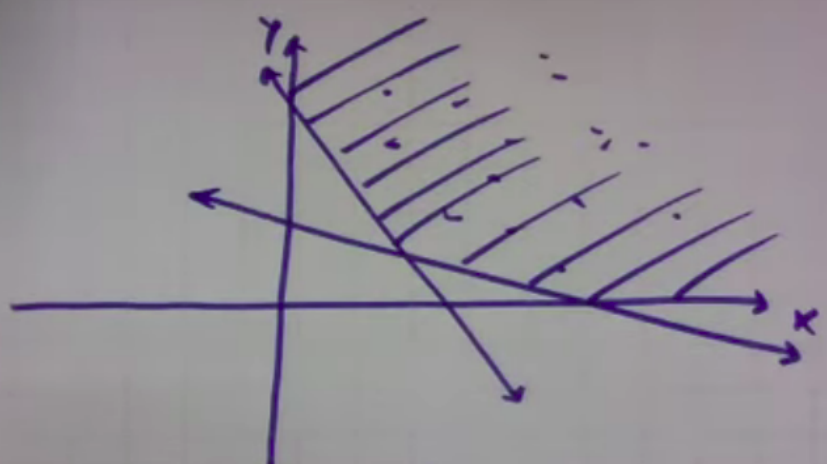
4. Optimization Problem:

A problem which seeks to maximize / minimize under certain constraints / condition.

5. Feasible Region :

OABC is called feasible region satisfied by given constraints.





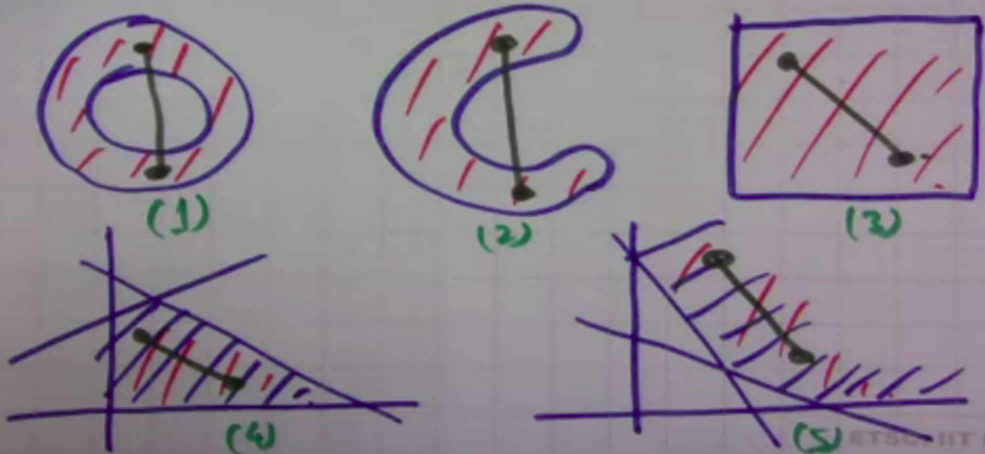
Unbounded feasible region.

6. Feasible solution :

$(\alpha, \beta) \in$ feasible region.

then (α, β) is called feasible solution for the given constraints simultaneously.

⊛ Every feasible region must be convex set.



⊛ Graphical Method of Solving a L.P.P.

The following theorems are fundamental in solving L.P.P.

Theorem 1:

Let R be the feasible region for an LPP and $Z = ax + by$ be the objective function, when Z has an optimal value (Max^{m} / Min^{m}), subject to the constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region.

Theorem 2:

Let R be the feasible region for an LPP and $Z = ax + by$ be the objective function. If R is bounded, then the objective function Z has both a maximum and a minimum value on R and each of these occurs at a corner point (vertex) of R.

Remark: If R is unbounded, then a max^{m} or a min^{m} value of the objective function may not exist, if it exist, it must occur at a corner point of R (By Th-1).

⊛ **Optimal Solution:** Any point in the feasible region that gives the optimal value of the objective function is called optimal sol^{n}.

Corner Point:

Corner point of a feasible region is a point in the region which is the intersection of two boundary lines.

Bounded Region:

A feasible region of a system of linear inequalities is said to be bounded if it can be enclosed within a circle.

Otherwise, it is called unbounded.

Corner Point Method:

It is method of solving LPP.

STEPS:

1. Find the feasible region of the LPP and determine its corner points either by inspection or by solving the two equations of the lines.

2. Evaluate $Z = ax + by$ at each corner point. Let M and m respectively denote the largest and smallest values.

3. (i) When the feasible region is bounded, M and m are the maximum and minimum values of Z.

(ii) In case, the feasible region is unbounded, we have

(a) M is the maximum value of Z, if the open half plane determined by $ax + by > M$ has no point in common with the feasible region.

Otherwise Z has no maximum value.