

3. Let  $S$  be the focus of the parabola  $y^2 = 8x$  and let  $PQ$  be the common chord of the circle  $x^2 + y^2 - 2x - 4y = 0$  and the given parabola. The area of the triangle  $PQS$  is (2012)

Solution: -

3. (4) We observe both parabola  $y^2 = 8x$  and circle  $x^2 + y^2 - 2x - 4y = 0$  pass through origin  
 $\therefore$  One end of common chord  $PQ$  is origin. Say  $P(0,0)$   
 Let  $Q$  be the point  $(2t^2, 4t)$ , then it will satisfy the equation of circle.  
 $\therefore 4t^4 + 16t^2 - 4t^2 - 16t = 0$   
 $\Rightarrow t^4 + 3t^2 - 4t = 0 \Rightarrow t(t^3 + 3t - 4) = 0$   
 $\Rightarrow t(t-1)(t^2 + t - 4) = 0 \Rightarrow t = 0$  or  $1$   
 For  $t = 0$ , we get point  $P$ , therefore  $t = 1$  gives point  $Q$  as  $(2, 4)$ .  
 We also observe here that  $P(0, 0)$  and  $Q(2, 4)$  are end points of diameter of the given circle and focus of the parabola is the point  $S(2, 0)$ .

$$\therefore \text{Area of } \Delta PQS = \frac{1}{2} \times PS \times QS = \frac{1}{2} \times 2 \times 4 = 4 \text{ sq. units}$$

