# Comprehension based questions: -

Let $a$, $r$, $s$, $t$ be nonzero real numbers. Let $P(at^2, 2at)$, $Q$, $R(ar^2, 2ar)$ and $S(as^2, 2as)$ be distinct points on the parabola $y^2 = 4ax$. Suppose that $PQ$ is the focal chord and lines $QR$ and $PK$ are parallel, where $K$ is the point $(2a, 0)$ ***(JEE Adv. 2014)***

**1.** The value of $r$ is

(a) $-\frac{1}{t}$ (b) $\frac{t^2+1}{t}$ (c) $\frac{1}{t}$ (d) $\frac{t^2-1}{t}$

**2.** If $st = 1$, then the tangent at $P$ and the normal at $S$ to the parabola meet at a point whose ordinate is

(a) $\frac{(t^2+1)^2}{2t^3}$ (b) $\frac{a(t^2+1)^2}{2t^3}$

(c) $\frac{a(t^2+1)^2}{t^3}$ (d) $\frac{a(t^2+2)^2}{t^3}$

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**Solution: -**

1. **(d)** $\because PQ$ is a focal chord, $Q\left(\frac{a}{t^2}, \frac{-2a}{t}\right)$

Also $QR \parallel PK \Rightarrow m_{QR} = m_{PK}$

$$\Rightarrow \frac{-\frac{2a}{t} - 2ar}{\frac{a}{t^2} - ar^2} = \frac{0 - 2at}{2a - at^2}$$

$$\Rightarrow \frac{-2a\left(\frac{1}{t} + r\right)}{a\left(\frac{1}{t} + r\right)\left(\frac{1}{t} - r\right)} = \frac{-2at}{a\left(2 - t^2\right)}$$

$$\Rightarrow 2 - t^2 = t\left(\frac{1}{t} - r\right)$$

$$\left[\because r \neq -\frac{1}{t} \text{ otherwise } Q \text{ will coincide with } R\right]$$

$$\Rightarrow 2 - t^2 = 1 - tr \Rightarrow r = \frac{t^2 - 1}{t}$$

**2.** **(b)** Tangent at $P$ is

$$ty = x + at^2 \qquad \text{....(i)}$$

Normal at S

$$sx + y = 2as + as^3 \qquad \text{....(ii)}$$

But given $st = 1 \Rightarrow s = \frac{1}{t}$

$$\therefore \frac{x}{t} + y = \frac{2a}{t} + \frac{a}{t^3}$$

$$\Rightarrow xt^2 = yt^3 = 2at^2 + a$$

Putting value of $x$ from equation (i) in above equation we get

$$\Rightarrow t^2\left(ty - at^2\right) + yt^3 = 2at^2 + a$$

$$\Rightarrow \left(t^3 + t^3\right)y - at^4 = 2at^2 + a$$

$$\Rightarrow 2t^3 y = a\left(t^4 + 2t^2 + 1\right)$$

$$y = \frac{a\left(t^4 + 2t^2 + 1\right)}{2t^3} = \frac{a\left(t^2 + 1\right)^2}{2t^3}$$

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