

**Solution** The correct choice is A. Indeed applying  $R_3 \rightarrow R_3 - \cos y R_1 + \sin y R_2$ , we get

$$\Delta = \begin{vmatrix} \cos x & -\sin x & 1 \\ \sin x & \cos x & 1 \\ 0 & 0 & \sin y - \cos y \end{vmatrix}.$$

Expanding along  $R_3$ , we have

$$\begin{aligned} \Delta &= (\sin y - \cos y) (\cos^2 x + \sin^2 x) \\ &= (\sin y - \cos y) = \sqrt{2} \left[ \frac{1}{\sqrt{2}} \sin y - \frac{1}{\sqrt{2}} \cos y \right] \\ &= \sqrt{2} \left[ \cos \frac{\pi}{4} \sin y - \sin \frac{\pi}{4} \cos y \right] = \sqrt{2} \sin \left( y - \frac{\pi}{4} \right) \end{aligned}$$

Hence  $-\sqrt{2} \leq \Delta \leq \sqrt{2}$ .

**Solution** Answer is 0. Apply  $R_2 \rightarrow R_2 - R_1$ ,  $R_3 \rightarrow R_3 - R_1$ .

**Sol.** 
$$\begin{vmatrix} y^2 z^2 & yz & y+z \\ z^2 x^2 & zx & z+x \\ x^2 y^2 & xy & x+y \end{vmatrix}$$

[Multiplying  $R_1, R_2, R_3$  by  $x, y, z$  respectively]

$$= \frac{1}{xyz} \begin{vmatrix} xy^2 z^2 & xyz & xy + xz \\ x^2 yz^2 & xyz & yz + xy \\ x^2 y^2 z & xyz & xz + yz \end{vmatrix}$$

[Taking  $(xyz)$  common from  $C_1$  and  $C_2$ ]

$$= \frac{1}{xyz} (xyz)^2 \begin{vmatrix} yz & 1 & xy + xz \\ xz & 1 & yz + xy \\ xy & 1 & xz + yz \end{vmatrix}$$

[Applying  $C_3 \rightarrow C_3 + C_1$ ]

$$= xyz \begin{vmatrix} yz & 1 & xy + yz + zx \\ xz & 1 & xy + yz + zx \\ xy & 1 & xy + yz + zx \end{vmatrix}$$

[Taking  $(xy + yz + zx)$  common from  $C_3$ ]

$$= xyz (xy + yz + zx) \begin{vmatrix} yz & 1 & 1 \\ xz & 1 & 1 \\ xy & 1 & 1 \end{vmatrix}$$

$$= 0 \quad [\because C_2 \text{ and } C_3 \text{ are identical}]$$

**Sol.** 
$$\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix}$$

[Applying  $C_1 \rightarrow C_1 + C_2 + C_3$ ]

$$= \begin{vmatrix} 2(y+z) & z & y \\ 2(z+x) & z+x & x \\ 2(y+x) & x & x+y \end{vmatrix}$$

$$= 2 \begin{vmatrix} y+z & z & y \\ z+x & z+x & x \\ x+y & x & x+y \end{vmatrix}$$

[Applying  $C_1 \rightarrow C_1 - C_2$ ]

$$= 2 \begin{vmatrix} y & z & y \\ 0 & z+x & x \\ y & x & x+y \end{vmatrix}$$

[Applying  $C_3 \rightarrow C_3 - C_1$ ]

$$= 2 \begin{vmatrix} y & z & 0 \\ 0 & z+x & x \\ y & x & x \end{vmatrix}$$

[Applying  $R_3 \rightarrow R_3 - R_1$ ]

$$= 2 \begin{vmatrix} y & z & 0 \\ 0 & z+x & x \\ 0 & x-z & x \end{vmatrix}$$

$$= 2y[(z+x)x - x(x-z)] = 2y[2xz] = 4xyz$$

**Sol.** 
$$\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

[Applying  $R_1 \rightarrow R_1 - R_2$  and  $R_2 \rightarrow R_2 - R_3$ ]

$$= \begin{vmatrix} a^2 - 1 & a - 1 & 0 \\ 2a - 2 & a - 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

[Taking  $(a - 1)$  common from  $R_1$  and  $R_2$ ]

$$(a - 1)^2 \begin{vmatrix} a + 1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

[Expanding along  $R_3$ ]

$$= (a - 1)^2 [1 \cdot (a + 1) - 2] = (a - 1)^3$$

**Sol.** We have 
$$\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$

[Applying  $R_1 \rightarrow R_1 + R_2 + R_3$ ]

$$\Rightarrow \begin{vmatrix} 12+x & 12+x & 12+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$

[Taking  $(12+x)$  common from  $R_1$ ]

$$\Rightarrow (12+x) \begin{vmatrix} 1 & 1 & 1 \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$

[Applying  $C_1 \rightarrow C_1 - C_3$  and  $C_2 \rightarrow C_2 - C_3$ ]

$$\Rightarrow (12+x) \begin{vmatrix} 0 & 0 & 1 \\ 0 & -2x & 4+x \\ 2x & 2x & 4-x \end{vmatrix} = 0$$

$$\Rightarrow (12+x)(0 - (-2x)(2x)) = 0$$

$$\Rightarrow (12+x)(4x^2) = 0$$

$$\therefore x = -12, 0$$

**Sol.** We have,  $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 + \cos A & 1 + \cos B & 1 + \cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{vmatrix} = 0$

[Applying  $C_1 \rightarrow C_1 - C_3$  and  $C_2 \rightarrow C_2 - C_3$ ]

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ \cos A - \cos C & \cos B - \cos C & 1 + \cos C \\ \cos^2 A + \cos A - \cos^2 C - \cos C & \cos^2 B + \cos B - \cos^2 C - \cos C & \cos^2 C + \cos C \end{vmatrix} = 0$$

[Taking  $(\cos A - \cos C)$  common from  $C_1$  and  $(\cos B - \cos C)$  common from  $C_2$ ]  
 $\Rightarrow (\cos A - \cos C)(\cos B - \cos C) \times$

$$\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 + \cos C \\ \cos A + \cos C + 1 & \cos B + \cos C + 1 & \cos^2 C + \cos C \end{vmatrix} = 0$$

[Applying  $C_1 \rightarrow C_1 - C_3$  and  $C_2 \rightarrow C_2 - C_3$ ]

$$\Rightarrow \begin{vmatrix} 0 & 0 & 1 \\ \cos A - \cos C & \cos B - \cos C & 1 + \cos C \\ \cos^2 A + \cos A - \cos^2 C - \cos C & \cos^2 B + \cos B - \cos^2 C - \cos C & \cos^2 C + \cos C \end{vmatrix} = 0$$

[Taking  $(\cos A - \cos C)$  common from  $C_1$  and  $(\cos B - \cos C)$  common from  $C_2$ ]  
 $\Rightarrow (\cos A - \cos C)(\cos B - \cos C) \times$

$$\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 + \cos C \\ \cos A + \cos C + 1 & \cos B + \cos C + 1 & \cos^2 C + \cos C \end{vmatrix} = 0$$

[Applying  $C_1 \rightarrow C_1 - C_2$ ]

$\Rightarrow (\cos A - \cos C)(\cos B - \cos C) \times$

$$\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 + \cos C \\ \cos A - \cos B & \cos B + \cos C + 1 & \cos^2 C + \cos C \end{vmatrix} = 0$$

$$\Rightarrow (\cos A - \cos C)(\cos B - \cos C)(\cos B - \cos A) = 0$$

$$\Rightarrow \cos A = \cos C \text{ or } \cos B = \cos C \text{ or } \cos B = \cos A$$

$$\Rightarrow A = C \text{ or } B = C \text{ or } B = A$$

Hence,  $\Delta ABC$  is an isosceles triangle.