

Example 11 If $x, y \in \mathbf{R}$, then the determinant $\Delta = \begin{vmatrix} \cos x & -\sin x & 1 \\ \sin x & \cos x & 1 \\ \cos(x+y) & -\sin(x+y) & 0 \end{vmatrix}$ lies

in the interval

(A) $[-\sqrt{2}, \sqrt{2}]$

(B) $[-1, 1]$

(C) $[-\sqrt{2}, 1]$

(D) $[-1, -\sqrt{2},]$

Example 12 If A, B, C are the angles of a triangle, then

$$\Delta = \begin{vmatrix} \sin^2 A & \cot A & 1 \\ \sin^2 B & \cot B & 1 \\ \sin^2 C & \cot C & 1 \end{vmatrix} = \dots\dots\dots$$

Using the properties of determinants in Exercises 7 to 9, prove that:

7. $\begin{vmatrix} y^2 z^2 & yz & y+z \\ z^2 x^2 & zx & z+x \\ x^2 y^2 & xy & x+y \end{vmatrix} = 0$

8. $\begin{vmatrix} y+z & z & y \\ z & z+x & x \\ y & x & x+y \end{vmatrix} = 4xyz$

9. $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$

13. If $\begin{bmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{bmatrix} = 0$, then find values of x.

16. Show that the ΔABC is an isosceles triangle if the determinant

$$\Delta = \begin{bmatrix} 1 & 1 & 1 \\ 1 + \cos A & 1 + \cos B & 1 + \cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{bmatrix} = 0 .$$