

1. Find the equation of the circle which touches the both axes in first quadrant and whose radius is a.

So, the equation of required circle is:

$$\begin{aligned} & (x - a)^2 + (y - a)^2 = a^2 \\ \Rightarrow & x^2 - 2ax + a^2 + y^2 - 2ay + a^2 = a^2 \\ \Rightarrow & x^2 + y^2 - 2ax - 2ay + a^2 = 0 \end{aligned}$$

4. Find the equation of the circle which touches x-axis and whose centre is (1, 2)

Sol: Given that, circle with centre (1,2) touches x-axis.

Radius of the circle is, $r = 2$

So, the equation of the required circle is:

$$\begin{aligned} & (x - 1)^2 + (y - 2)^2 = 2^2 \\ \Rightarrow & x^2 - 2x + 1 + y^2 - 4y + 4 = 4 \\ \Rightarrow & x^2 + y^2 - 2x - 4y + 1 = 0 \end{aligned}$$

6. Find the equation of a circle which touches both the axes and the line $3x - 4y + 8 = 0$ and lies in the third quadrant. [Hint: Let a be the radius of the circle, then $(-a, -a)$ will be centre and perpendicular distance from the centre to the given line gives the radius of the circle.]

$$\begin{aligned} \therefore & \frac{a + 8}{5} = \pm a \\ \Rightarrow & a + 8 = 5a \text{ or } a + 8 = -5a \\ \Rightarrow & a = 2 \text{ or } a = -4/3 \\ \therefore & a = 2 \end{aligned}$$

So, the equation of the required circle is:

$$\begin{aligned} & (x + 2)^2 + (y + 2)^2 = 2^2 \\ \Rightarrow & x^2 + y^2 + 4x + 4y + 4 = 0 \end{aligned}$$

48. Equation of a circle which passes through (3, 6) and touches the axes is

- (a) $x^2 + y^2 + 6x + 6y + 3 = 0$ (b) $x^2 + y^2 - 6x - 6y - 9 = 0$
(c) $x^2 + y^2 - 6x - 6y + 9 = 0$ (d) none of these

Sol. (c) Given that the circle touches both axes.

Therefore, equation of the circle is: $(x - a)^2 + (y - a)^2 = a^2$

Circle passes through the point (3, 6).

$$\therefore (3 - a)^2 + (6 - a)^2 = a^2$$

$$\Rightarrow a^2 - 18a + 45 = 0 \Rightarrow (a - 3)(a - 15) = 0$$

$$\therefore a = 3, a = 15$$

For $a = 3$, the equation of circle is:

$$(x - 3)^2 + (y - 3)^2 = 9$$

$$\Rightarrow x^2 + y^2 - 6x - 6y + 9 = 0$$