According to Stefan's law of radiation, a black body radiates energy σT^4 from its unit surface area every second where T is the surface temperature of the black body and $\sigma = 5.67 \times 10^{-8}$ W/m^2K^4 is known as Stefan's constant. A nuclear weapon may be thought of as a ball of radius 0.5 m. When detonated, it reaches temperature of 10^6K and can be treated as a black body.

(a) Estimate the power it radiates.

(b) If surrounding has water at $30^{\circ}C$, how much water can 10% of the energy produced evaporate in 1 s?

$$\left[S_w = 4186.0\,\text{J/kg K} \quad \text{and } L_v = 22.6 \times 10^5\,\text{J/kg}\right]$$

(c) If all this energy U is in the form of radiation, corresponding momentum is $p = U/c$. How much momentum per unit time does it impart on unit area at a distance of 1 km?

Given ; $\sigma = 5.67 \times 10^{-8}\ W/m^2/K^4$, $r = 0.5\,m$, $T = 10^6\,K$

$E = \sigma T^4$ per second per sq. m

(a) Total P = Total E radiated from all area 'A' per sec.

$= \sigma A T^4$

$= (5.67 \times 10^{-8})[4\pi(0.5)^2](10^6)^4$

$P \approx 18 \times 10^{16}$ watt

(b) 10% of total $P \Rightarrow \frac{10}{100} \times 18 \times 10^{16} \Rightarrow 1.8 \times 10^{16}$ J/s

$S_w = 4186$ J/Kg K ; $L_v = 22.5 \times 10^5$ J/Kg

Now, E required by 'm' kg of water at 30°C to evaporate at 100°C = E required to heat water from 30°C to 100°C + E required to evaporate water to vapour.

$$= mS_w \Delta T + mL_v$$

$$1.8 \times 10^{16} = m[4180(100-30) + 22.6 \times 10^5]$$

$$\Rightarrow m = \frac{1.8 \times 10^{16}}{25.5 \times 10^5}$$

$$\Rightarrow \boxed{m \approx 7 \times 10^9 \text{ Kg}}$$

(c) Now, momentum per unit time $(p') = \frac{U}{c}$

[$c = 3 \times 10^8$ m/s (speed of light)]

$$\Rightarrow p' = \frac{1.8 \times 10^{17}}{3 \times 10^8}$$

$$\Rightarrow \boxed{p' = 6 \times 10^9 \text{ kg-m/s}^2}$$

Hence, 'p' per unit time per unit area at a distance of 1 Km $\Rightarrow \frac{p'}{A}$; $\{A = 4\pi R^2$, $R = 10^3$ m. here$\}$

$$\Rightarrow \frac{6 \times 10^9}{4\pi (10^3)^2}$$

$$= \boxed{47.77 \frac{\text{Kg-m/s}}{\text{m}^2}}$$