

Fig. 11.2 Pressure versus temperature of a low density gas kept at constant volume.

relationship. Notice that since $PV = \text{constant}$ and $V/T = \text{constant}$ for a given quantity of gas, then PV/T should also be a constant. This relationship is known as ideal gas law. It can be written in a more general form that applies not just to a given quantity of a single gas but to any quantity of any low-density gas and is known as **ideal-gas equation**:

$$\frac{PV}{T} = \mu R$$

$$\text{or } PV = \mu RT \quad (11.2)$$

where, μ is the number of moles in the sample of gas and R is called universal gas constant:

$$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

In Eq. 11.2, we have learnt that the pressure and volume are directly proportional to temperature: $PV \propto T$. This relationship allows a gas to be used to measure temperature in a constant volume gas thermometer. Holding the volume of a gas constant, it gives $P \propto T$. Thus, with a constant-volume gas thermometer, temperature is read in terms of pressure. A plot of pressure versus temperature gives a straight line in this case, as shown in Fig. 11.2.

However, measurements on real gases deviate from the values predicted by the ideal gas law at low temperature. But the relationship is linear over a large temperature range, and it looks as though the pressure might reach zero with decreasing temperature if the gas continued to be a gas. The absolute minimum temperature for an ideal gas, therefore, inferred by extrapolating the straight line to the axis, as in Fig. 11.3. This temperature is found to be -273.15°C and is designated as **absolute zero**. Absolute zero is the foundation of the Kelvin temperature scale or absolute scale temperature

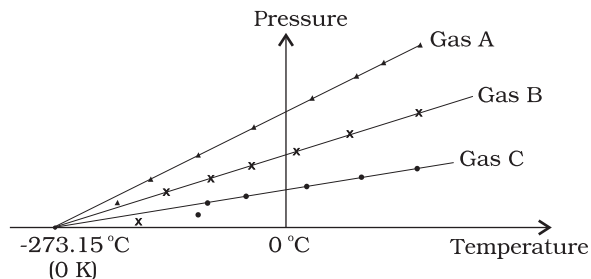


Fig. 11.3 A plot of pressure versus temperature and extrapolation of lines for low density gases indicates the same absolute zero temperature.

named after the British scientist Lord Kelvin. On this scale, -273.15°C is taken as the zero point, that is 0 K (Fig. 11.4).

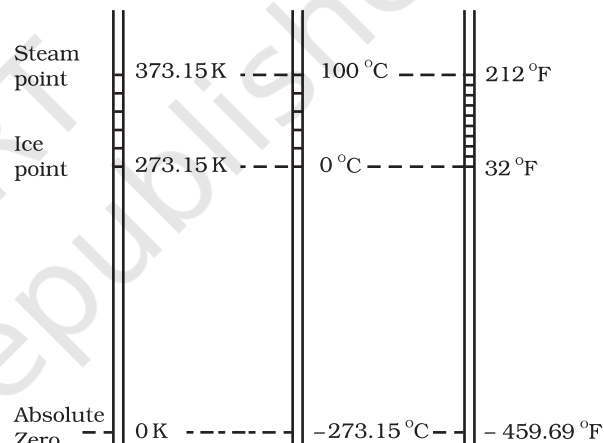


Fig. 11.4 Comparison of the Kelvin, Celsius and Fahrenheit temperature scales.

The size of unit in Kelvin and Celsius temperature scales is the same. So, temperature on these scales are related by

$$T = t_c + 273.15 \quad (11.3)$$

11.5 THERMAL EXPANSION

You may have observed that sometimes sealed bottles with metallic lids are so tightly screwed that one has to put the lid in hot water for some time to open it. This would allow the metallic lid to expand, thereby loosening it to unscrew easily. In case of liquids, you may have observed that mercury in a thermometer rises, when the thermometer is put in slightly warm water. If we take out the thermometer from the warm

water the level of mercury falls again. Similarly, in case of gases, a balloon partially inflated in a cool room may expand to full size when placed in warm water. On the other hand, a fully inflated balloon when immersed in cold water would start shrinking due to contraction of the air inside.

It is our common experience that most substances expand on heating and contract on cooling. A change in the temperature of a body causes change in its dimensions. The increase in the dimensions of a body due to the increase in its temperature is called thermal expansion. The expansion in length is called **linear expansion**. The expansion in area is called **area expansion**. The expansion in volume is called **volume expansion** (Fig. 11.5).

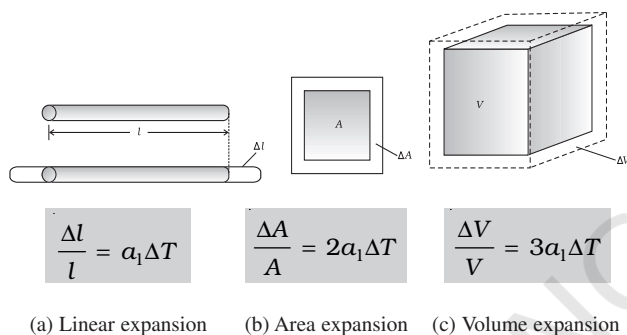


Fig. 11.5 Thermal Expansion.

If the substance is in the form of a long rod, then for small change in temperature, ΔT , the fractional change in length, $\Delta l/l$, is directly proportional to ΔT .

$$\frac{\Delta l}{l} = \alpha_1 \Delta T \quad (11.4)$$

where α_1 is known as the **coefficient of linear expansion** (or linear expansivity) and is characteristic of the material of the rod. In Table 11.1, typical average values of the coefficient of linear expansion for some material in the temperature range 0 °C to 100 °C are given. From this Table, compare the value of α_1 for glass and copper. We find that copper expands about five times more than glass for the same rise in temperature. Normally, metals expand more and have relatively high values of α_1 .

Table 11.1 Values of coefficient of linear expansion for some material

Material	α_1 (10^{-5} K^{-1})
Aluminium	2.5
Brass	1.8
Iron	1.2
Copper	1.7
Silver	1.9
Gold	1.4
Glass (pyrex)	0.32
Lead	0.29

Similarly, we consider the fractional change in volume, $\frac{\Delta V}{V}$, of a substance for temperature change ΔT and define the **coefficient of volume expansion (or volume expansivity)**, α_v as

$$\alpha_v = \left(\frac{\Delta V}{V} \right) \frac{1}{\Delta T} \quad (11.5)$$

Here α_v is also a characteristic of the substance but is not strictly a constant. It depends in general on temperature (Fig 11.6). It is seen that α_v becomes constant only at a high temperature.

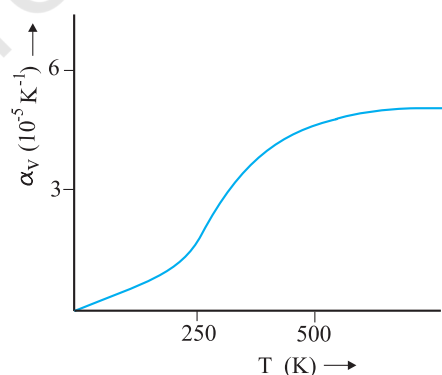


Fig. 11.6 Coefficient of volume expansion of copper as a function of temperature.

Table 11.2 gives the values of coefficient of volume expansion of some common substances in the temperature range 0–100 °C. You can see that thermal expansion of these substances (solids and liquids) is rather small, with material,

like pyrex glass and invar (a special iron-nickel alloy) having particularly low values of α_v . From this Table we find that the value of α_v for alcohol (ethanol) is more than mercury and expands more than mercury for the same rise in temperature.

Table 11.2 Values of coefficient of volume expansion for some substances

Material	α_v (K^{-1})
Aluminium	7×10^{-5}
Brass	6×10^{-5}
Iron	3.55×10^{-5}
Paraffin	58.8×10^{-5}
Glass (ordinary)	2.5×10^{-5}
Glass (pyrex)	1×10^{-5}
Hard rubber	2.4×10^{-4}
Invar	2×10^{-6}
Mercury	18.2×10^{-5}
Water	20.7×10^{-5}
Alcohol (ethanol)	110×10^{-5}

Water exhibits an anomalous behaviour; it contracts on heating between $0^\circ C$ and $4^\circ C$. The volume of a given amount of water decreases as it is cooled from room temperature, until its temperature reaches $4^\circ C$, [Fig. 11.7(a)]. Below $4^\circ C$, the volume increases, and therefore, the density decreases [Fig. 11.7(b)].

This means that water has the maximum density at $4^\circ C$. This property has an important environmental effect: bodies of water, such as

lakes and ponds, freeze at the top first. As a lake cools toward $4^\circ C$, water near the surface loses energy to the atmosphere, becomes denser, and sinks; the warmer, less dense water near the bottom rises. However, once the colder water on top reaches temperature below $4^\circ C$, it becomes less dense and remains at the surface, where it freezes. If water did not have this property, lakes and ponds would freeze from the bottom up, which would destroy much of their animal and plant life.

Gases, at ordinary temperature, expand more than solids and liquids. For liquids, the coefficient of volume expansion is relatively independent of the temperature. However, for gases it is dependent on temperature. For an ideal gas, the coefficient of volume expansion at constant pressure can be found from the ideal gas equation:

$$PV = \mu RT$$

At constant pressure

$$P\Delta V = \mu R \Delta T$$

$$\frac{\Delta V}{V} = \frac{\Delta T}{T}$$

$$\text{i.e., } \alpha_v = \frac{1}{T} \text{ for ideal gas} \quad (11.6)$$

At $0^\circ C$, $\alpha_v = 3.7 \times 10^{-3} K^{-1}$, which is much larger than that for solids and liquids. Equation (11.6) shows the temperature dependence of α_v ; it decreases with increasing temperature. For a gas at room temperature and constant pressure, α_v is about $3300 \times 10^{-6} K^{-1}$, as

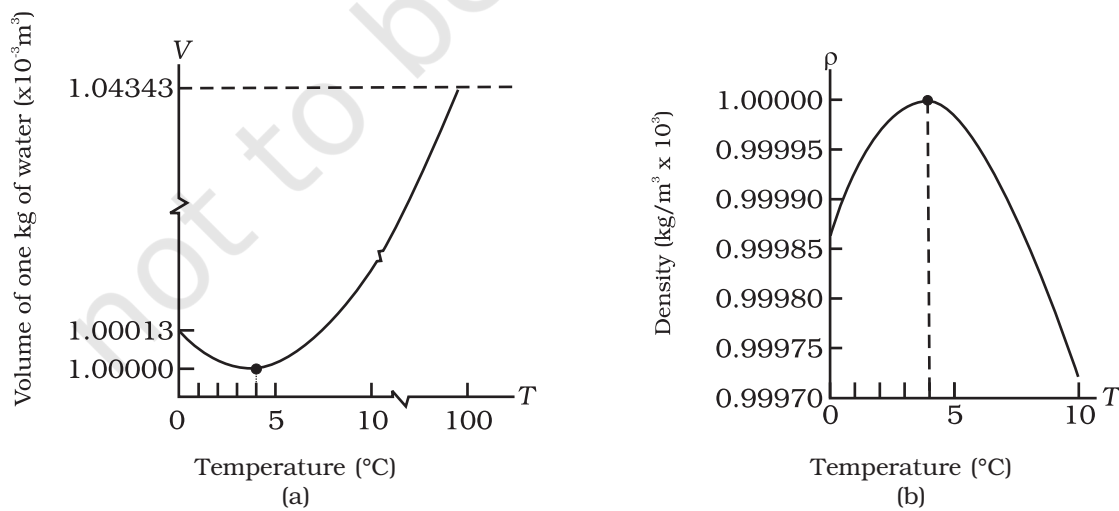


Fig. 11.7 Thermal expansion of water.

much as order(s) of magnitude larger than the coefficient of volume expansion of typical liquids.

There is a simple relation between the coefficient of volume expansion (α_v) and coefficient of linear expansion (α_l). Imagine a cube of length, l , that expands equally in all directions, when its temperature increases by ΔT . We have

$$\Delta l = \alpha_l l \Delta T$$

so, $\Delta V = (l + \Delta l)^3 - l^3 \approx 3l^2 \Delta l$ (11.7)

In Equation (11.7), terms in $(\Delta l)^2$ and $(\Delta l)^3$ have been neglected since Δl is small compared to l . So

$$\Delta V = \frac{3V \Delta l}{l} = 3V \alpha_l \Delta T$$
 (11.8)

which gives

$$\alpha_v = 3\alpha_l$$
 (11.9)

What happens by preventing the thermal expansion of a rod by fixing its ends rigidly? Clearly, the rod acquires a compressive strain due to the external forces provided by the rigid support at the ends. The corresponding stress set up in the rod is called **thermal stress**. For example, consider a steel rail of length 5 m and area of cross-section 40 cm² that is prevented from expanding while the temperature rises by 10 °C. The coefficient of linear expansion of steel is $\alpha_{l(\text{steel})} = 1.2 \times 10^{-5} \text{ K}^{-1}$. Thus, the compressive strain is

$$\frac{\Delta l}{l} = \alpha_{l(\text{steel})} \Delta T = 1.2 \times 10^{-5} \times 10 = 1.2 \times 10^{-4}$$

Young's modulus of steel is $Y_{(\text{steel})} = 2 \times 10^{11} \text{ N m}^{-2}$. Therefore, the thermal stress developed is

$$\frac{\Delta F}{A} = Y_{\text{steel}} \left(\frac{\Delta l}{l} \right) = 2.4 \times 10^7 \text{ N m}^{-2}$$

which corresponds to an external force of

$$\Delta F = AY_{\text{steel}} \left(\frac{\Delta l}{l} \right) = 2.4 \times 10^7 \times 40 \times 10^{-4} \approx 10^5 \text{ N}$$

If two such steel rails, fixed at their outer ends, are in contact at their inner ends, a force of this magnitude can easily bend the rails.

► **Example 11.1** Show that the coefficient of area expansion, $(\Delta A/A)/\Delta T$, of a rectangular sheet of the solid is twice its linear expansivity, α_l .

Answer

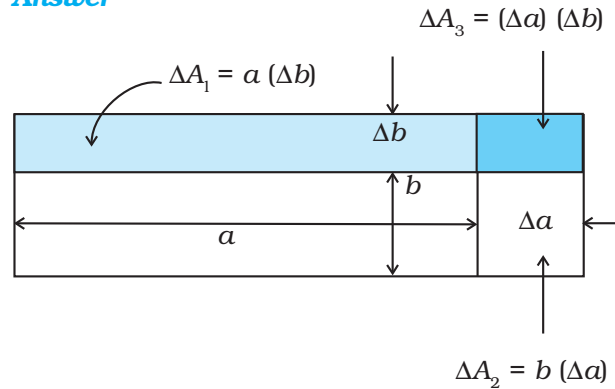


Fig. 11.8

Consider a rectangular sheet of the solid material of length a and breadth b (Fig. 11.8). When the temperature increases by ΔT , a increases by $\Delta a = \alpha_l a \Delta T$ and b increases by $\Delta b = \alpha_l b \Delta T$. From Fig. 11.8, the increase in area

$$\begin{aligned} \Delta A &= \Delta A_1 + \Delta A_2 + \Delta A_3 \\ \Delta A &= a \Delta b + b \Delta a + (\Delta a)(\Delta b) \\ &= a \alpha_l b \Delta T + b \alpha_l a \Delta T + (\alpha_l)^2 ab (\Delta T)^2 \\ &= \alpha_l ab \Delta T (2 + \alpha_l \Delta T) = \alpha_l A \Delta T (2 + \alpha_l \Delta T) \end{aligned}$$

Since $\alpha_l \approx 10^{-5} \text{ K}^{-1}$, from Table 11.1, the product $\alpha_l \Delta T$ for fractional temperature is small in comparison with 2 and may be neglected. Hence,

$$\left(\frac{\Delta A}{A} \right) \frac{1}{\Delta T} \approx 2\alpha_l$$

► **Example 11.2** A blacksmith fixes iron ring on the rim of the wooden wheel of a horse cart. The diameter of the rim and the iron ring are 5.243 m and 5.231 m, respectively at 27 °C. To what temperature should the ring be heated so as to fit the rim of the wheel?

Answer

Given, $T_1 = 27 \text{ °C}$
 $L_{r1} = 5.231 \text{ m}$
 $L_{r2} = 5.243 \text{ m}$

So, $L_{r2} = L_{r1} [1 + \alpha_l (T_2 - T_1)]$
 $5.243 \text{ m} = 5.231 \text{ m} [1 + 1.20 \times 10^{-5} \text{ K}^{-1} (T_2 - 27 \text{ °C})]$
 or $T_2 = 218 \text{ °C}$.