4 JEE Main 2021 (Online) 27th August Evening Shift

MCQ (Single Correct Answer)

Let $A = \begin{pmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{pmatrix}$, where [t] denotes the greatest integer less than or equal to t. If det(A) = 192, then the set of values of x is the interval :

A [68, 69)

B [62, 63)

C [65, 66)

D [60, 61)

Explanation

$$\begin{vmatrix} [x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4] \end{vmatrix} = 192$$

$R_1 \to R_1 - R_3$ & $R_2 \to R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ [x] & [x]+2 & [x]+4 \end{bmatrix} = 192$$

$$2[x] + 6 + [x] = 192 \Rightarrow [x] = 62$$

3 JEE Main 2021 (Online) 25th July Evening Shift
MCQ (Single Correct Answer)

The number of distinct real roots

of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$ is :

A 4

B 1

C 2

D 3

Explanation

$$\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0,\ -\frac{\pi}{4} \le x \le \frac{\pi}{4}$$

Apply : $R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$\Rightarrow \begin{vmatrix} \sin x - \cos x & \cos x - \sin x & 0 \\ 0 & \sin x - \cos x & \cos x - \sin x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$$\Rightarrow (\sin x - \cos x)^2 \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$$

$$\Rightarrow (\sin x - \cos x)^2 (\sin x + 2\cos x) = 0$$

$$\therefore\ x = \frac{\pi}{4}$$

2 JEE Main 2021 (Online) 17th March Evening Shift
MCQ (Single Correct Answer)

If x, y, z are in arithmetic progression with common difference d, x $\neq$ 3d, and the determinant of the matrix $\begin{bmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix}$ is zero, then the value of k^2 is :

A 72

B 12

C 36

D 6

Explanation

$$\begin{vmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{vmatrix} = 0$$

$$R_1 \to R_1 + R_3 - 2R_2$$

$$\Rightarrow \begin{vmatrix} 0 & 4\sqrt{2} - k - 10\sqrt{2} & 0 \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{vmatrix} = 0$$ { $\because$ 2y = x + z}

$$\Rightarrow (k - 6\sqrt{2})(4z - 5y) = 0$$

$\Rightarrow$ k = $6\sqrt{2}$ or 4z = 5y (Not possible $\because$ x, y, z in A.P.)

So, k^2 = 72

$\therefore$ Option (A)

4 JEE Main 2021 (Online) 26th February Morning Shift

MCQ (Single Correct Answer)

The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is :

A -2

B 0

C (a + 2)(a + 3)(a + 4)

D (a + 1)(a + 2)(a + 3)

Explanation

Given, $\Delta = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Delta = \begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3-a-1) & 1 & 0 \\ a^2+7a+12-a^2-3a-2 & 2 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} a^2+3a+2 & a+2 & 1 \\ 2(a+2) & 1 & 0 \\ 4a+10 & 2 & 0 \end{vmatrix}$$

$$= 4(a+2) - 4a - 10$$

$$= 4a + 8 - 4a - 10 = -2$$

4 JEE Main 2021 (Online) 25th February Evening Shift

MCQ (Single Correct Answer)

Let A be a 3 $\times$ 3 matrix with det(A) = 4. Let R_i denote the i^{th} row of A. If a matrix B is obtained by performing the operation $R_2 \to 2R_2 + 5R_3$ on 2A, then det(B) is equal to :

A 64

B 16

C 128

D 80

Explanation

$$A = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

$$2A = \begin{bmatrix} 2R_{11} & 2R_{12} & 2R_{13} \\ 2R_{21} & 2R_{22} & 2R_{23} \\ 2R_{31} & 2R_{32} & 2R_{33} \end{bmatrix}$$

$$R_2 \to 2R_2 + 5R_3$$

$$B = \begin{bmatrix} 2R_{11} & 2R_{12} & 2R_{13} \\ 4R_{21} + 10R_{31} & 4R_{22} + 10R_{32} & 4R_{23} + 10R_{33} \\ 2R_{31} & 2R_{32} & 2R_{33} \end{bmatrix}$$

$$R_2 \to R_2 - 5R_3$$

$$B = \begin{bmatrix} 2R_{11} & 2R_{12} & 2R_{13} \\ 4R_{21} & 4R_{22} & 4R_{23} \\ 2R_{31} & 2R_{32} & 2R_{33} \end{bmatrix}$$

$$|B| = \begin{bmatrix} 2R_{11} & 2R_{12} & 2R_{13} \\ 4R_{21} & 4R_{22} & 4R_{23} \\ 2R_{31} & 2R_{32} & 2R_{33} \end{bmatrix}$$

$$|B| = 2 \times 2 \times 4 \begin{vmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{vmatrix}$$

$$= 16 \times 4$$

$$= 64$$