Practice Questions

Q1.

Page-70

Example 3 Without expanding, show that

$$\Delta = \begin{vmatrix} \operatorname{cosec}^2\theta & \cot^2\theta & 1 \\ \cot^2\theta & \operatorname{cosec}^2\theta & -1 \\ 42 & 40 & 2 \end{vmatrix} = 0.$$

Solution Applying $C_1 \to C_1 - C_2 - C_3$, we have

$$\Delta = \begin{vmatrix} \operatorname{cosec}^2\theta - \cot^2\theta - 1 & \cot^2\theta & 1 \\ \cot^2\theta - \operatorname{cosec}^2\theta + 1 & \operatorname{cosec}^2\theta & -1 \\ 0 & 40 & 2 \end{vmatrix} = \begin{vmatrix} 0 & \cot^2\theta & 1 \\ 0 & \operatorname{cosec}^2\theta & -1 \\ 0 & 40 & 2 \end{vmatrix} = 0$$

Q2.

Page-71

Example 5 If $\Delta = \begin{vmatrix} 0 & b-a & c-a \\ a-b & 0 & c-b \\ a-c & b-c & 0 \end{vmatrix}$, then show that Δ is equal to zero.

Solution Interchanging rows and columns, we get $\Delta = \begin{vmatrix} 0 & a-b & a-c \\ b-a & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix}$

Taking '–1' common from R_1, R_2 and R_3, we get

$$\Delta = (-1)^3 \begin{vmatrix} 0 & b-a & c-a \\ a-b & 0 & c-b \\ a-c & b-c & 0 \end{vmatrix} = -\Delta$$

$$\Rightarrow \quad 2\Delta = 0 \quad \text{or} \quad \Delta = 0$$

Q3.

Page-71,72

Example 7 If $x = -4$ is a root of $\Delta = \begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$, then find the other two roots.

Solution Applying $R_1 \rightarrow (R_1 + R_2 + R_3)$, we get

$$\begin{vmatrix} x+4 & x+4 & x+4 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix}.$$

Taking $(x + 4)$ common from R_1, we get

$$\Delta = (x+4)\begin{vmatrix} 1 & 1 & 1 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 - C_1$, $C_3 \rightarrow C_3 - C_1$, we get

$$\Delta = (x+4)\begin{vmatrix} 1 & 0 & 0 \\ 1 & x-1 & 0 \\ 3 & -1 & x-3 \end{vmatrix}.$$

Expanding along R_1,

$\Delta = (x + 4) [(x - 1) (x - 3) - 0]$. Thus, $\Delta = 0$ implies

$$x = -4, 1, 3$$

Q4.

Page-72,73

Example 8 In a triangle ABC, if

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix} = 0,$$

then prove that ΔABC is an isoceles triangle.

Solution Let $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ \sin A+\sin^2 A & \sin B+\sin^2 B & \sin C+\sin^2 C \end{vmatrix}$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 1+\sin A & 1+\sin B & 1+\sin C \\ -\cos^2 A & -\cos^2 B & -\cos^2 C \end{vmatrix} \quad R_3 \to R_3 - R_2$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1+\sin A & \sin B - \sin A & \sin C - \sin B \\ -\cos^2 A & \cos^2 A - \cos^2 B & \cos^2 B - \cos^2 C \end{vmatrix} . \ (C_3 \to C_3 - C_2 \text{ and } C_2 \to C_2 - C_1)$$

Expanding along R_1, we get

$$\Delta = (\sin B - \sin A)(\sin^2 C - \sin^2 B) - (\sin C - \sin B)(\sin^2 B - \sin^2 A)$$

$$= (\sin B - \sin A)(\sin C - \sin B)(\sin C - \sin A) = 0$$

$\Rightarrow$ either $\sin B - \sin A = 0$ or $\sin C - \sin B$ or $\sin C - \sin A = 0$

$\Rightarrow$ $A = B$ or $B = C$ or $C = A$

i.e. triangle ABC is isoceles.

Q5. Objective Type

Example 10 Let $\Delta = \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix}$ and $\Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix}$, then

(A) $\Delta_1 = -\Delta$ (B) $\Delta \neq \Delta_1$

(C) $\Delta - \Delta_1 = 0$ (D) None of these

Solution (C) is the correct answer since $\Delta_1 = \begin{vmatrix} A & B & C \\ x & y & z \\ zy & zx & xy \end{vmatrix} = \begin{vmatrix} A & x & yz \\ B & y & zx \\ C & z & xy \end{vmatrix}$

$$= \frac{1}{xyz} \begin{vmatrix} Ax & x^2 & xyz \\ By & y^2 & xyz \\ Cz & z^2 & xyz \end{vmatrix} = \frac{xyz}{xyz} \begin{vmatrix} Ax & x^2 & 1 \\ By & y^2 & 1 \\ Cz & z^2 & 1 \end{vmatrix} = \Delta$$

Q6.

Example 13 The determinant $\Delta = \begin{vmatrix} \sqrt{23}+\sqrt{3} & \sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{46} & 5 & \sqrt{10} \\ 3+\sqrt{115} & \sqrt{15} & 5 \end{vmatrix}$ is equal to ....

Solution Answer is 0.Taking $\sqrt{5}$ common from C_2 and C_3 and applying $C_1 \to C_3 - \sqrt{3}\ C_2$, we get the desired result.