In the following sections, we shall obtain the equations of each of these conic sections in standard form by defining them based on geometric properties.

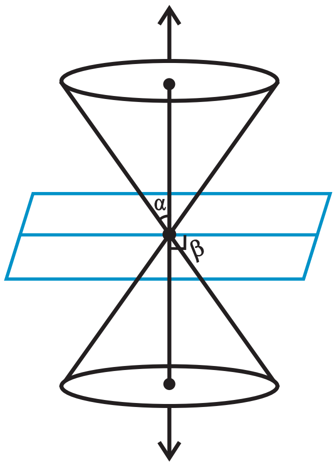


Fig 11. 8

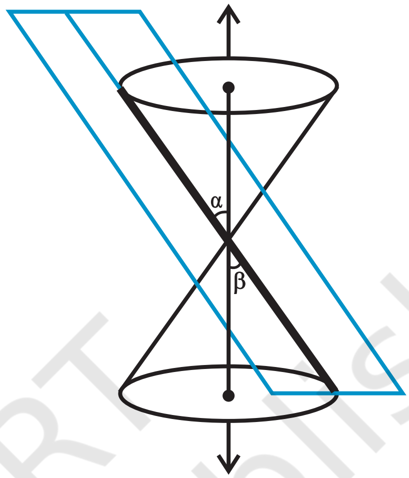
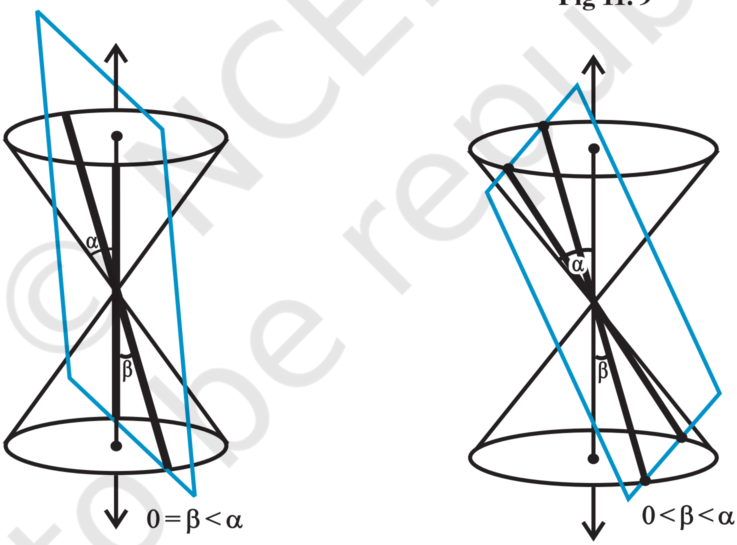


Fig 11. 9



(a) Fig 11. 10 (b)

11.3 Circle

Definition 1 A circle is the set of all points in a plane that are equidistant from a fixed point in the plane.

The fixed point is called the *centre of the circle* and the distance from the centre to a point on the circle is called the *radius* of the circle (Fig 11.11).

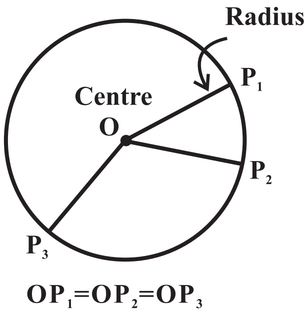


Fig 11. 11

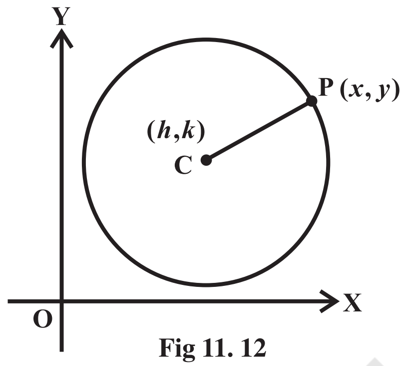


Fig 11. 12

The equation of the circle is simplest if the centre of the circle is at the origin. However, we derive below the equation of the circle with a given centre and radius (Fig 11.12).

Given C (h, k) be the centre and r the radius of circle. Let P(x, y) be any point on the circle (Fig11.12). Then, by the definition, $| CP | = r$. By the distance formula, we have

$$\sqrt{(x-h)^2+(y-k)^2}=r$$

i.e. $$(x-h)^2+(y-k)^2=r^2$$

This is the required equation of the circle with centre at (h,k) and radius r .

Example 1 Find an equation of the circle with centre at (0,0) and radius r.

Solution Here $h = k = 0$. Therefore, the equation of the circle is $x^2 + y^2 = r^2$.

Example 2 Find the equation of the circle with centre (–3, 2) and radius 4.

Solution Here $h = -3$, $k = 2$ and $r = 4$. Therefore, the equation of the required circle is

$$(x + 3)^2 + (y - 2)^2 = 16$$

Example 3 Find the centre and the radius of the circle $x^2 + y^2 + 8x + 10y - 8 = 0$

Solution The given equation is

$$(x^2 + 8x) + (y^2 + 10y) = 8$$

Now, completing the squares within the parenthesis, we get

$$(x^2 + 8x + 16) + (y^2 + 10y + 25) = 8 + 16 + 25$$

i.e. $$(x + 4)^2 + (y + 5)^2 = 49$$

i.e. $$\{x - (-4)\}^2 + \{y - (-5)\}^2 = 7^2$$

Therefore, the given circle has centre at (– 4, –5) and radius 7.

Example 4 Find the equation of the circle which passes through the points (2, – 2), and (3,4) and whose centre lies on the line $x + y = 2$.

Solution Let the equation of the circle be $(x - h)^2 + (y - k)^2 = r^2$.

Since the circle passes through (2, – 2) and (3,4), we have

$$(2 - h)^2 + (-2 - k)^2 = r^2 \quad \ldots (1)$$

and $$(3 - h)^2 + (4 - k)^2 = r^2 \quad \ldots (2)$$

Also since the centre lies on the line $x + y = 2$, we have

$$h + k = 2 \quad \ldots (3)$$

Solving the equations (1), (2) and (3), we get

$$h = 0.7, \quad k = 1.3 \quad \text{and } r^2 = 12.58$$

Hence, the equation of the required circle is

$$(x - 0.7)^2 + (y - 1.3)^2 = 12.58.$$

EXERCISE 11.1

In each of the following Exercises 1 to 5, find the equation of the circle with

1. centre (0,2) and radius 2

2. centre (–2,3) and radius 4

3. centre $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $\frac{1}{12}$

4. centre (1,1) and radius $\sqrt{2}$

5. centre $(-a, -b)$ and radius $\sqrt{a^2 - b^2}$.

In each of the following Exercises 6 to 9, find the centre and radius of the circles.

6. $(x + 5)^2 + (y - 3)^2 = 36$

7. $x^2 + y^2 - 4x - 8y - 45 = 0$

8. $x^2 + y^2 - 8x + 10y - 12 = 0$

9. $2x^2 + 2y^2 - x = 0$

10. Find the equation of the circle passing through the points (4,1) and (6,5) and whose centre is on the line $4x + y = 16$.

11. Find the equation of the circle passing through the points (2,3) and (–1,1) and whose centre is on the line $x - 3y - 11 = 0$.

12. Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2,3).

13. Find the equation of the circle passing through (0,0) and making intercepts a and b on the coordinate axes.

14. Find the equation of a circle with centre (2,2) and passes through the point (4,5).

15. Does the point (–2.5, 3.5) lie inside, outside or on the circle $x^2 + y^2 = 25$?