

1 JEE Main 2021 (Online) 26th August Morning Shift

Numerical

The locus of a point, which moves such that the sum of squares of its distances from the points $(0, 0)$, $(1, 0)$, $(0, 1)$, $(1, 1)$ is 18 units, is a circle of diameter d . Then d^2 is equal to _____.

Answer

Correct Answer is 16

Explanation

Let point $P(x, y)$

$A(0, 0)$, $B(1, 0)$, $C(0, 1)$, $D(1, 1)$

$$(PA)^2 + (PB)^2 + (PC)^2 + (PD)^2 = 18$$

$$x^2 + y^2 + x^2 + (y - 1)^2 + (x - 1)^2 + y^2 + (x - 1)^2 + (y - 1)^2 = 18$$

$$\Rightarrow 4(x^2 + y^2) - 4y - 4x = 14$$

$$\Rightarrow x^2 + y^2 - x - y - \frac{7}{2} = 0$$

$$d = 2\sqrt{\frac{1}{4} + \frac{1}{4} + \frac{7}{2}}$$

$$\Rightarrow d^2 = 16$$

Question 1: Let the tangents drawn from the origin to the circle $x^2 + y^2 - 8x - 4y + 16 = 0$ touch it at the point A and B. The $(AB)^2$ is equal to

- (a) $32/5$ (b) $64/5$ (c) $52/5$
 (d) $56/5$

Answer: (b)

Solution:

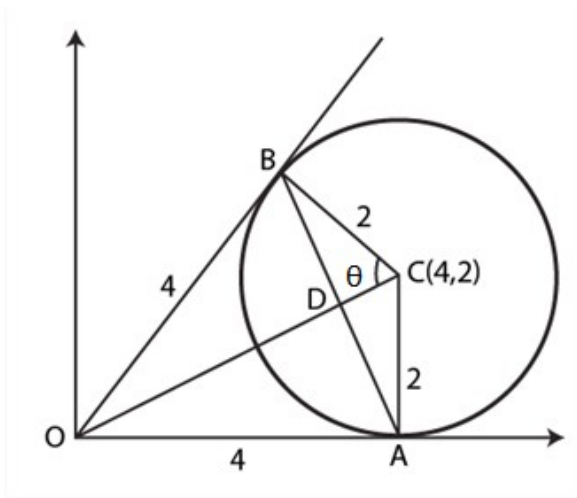
$$x^2 + y^2 - 8x - 4y + 16 = 0$$

Rearranging above equation, we get

$$(x - 4)^2 + (y - 2)^2 = 4$$

Centre = (4, 2) and

Radius = 2



$$OA = OB = 4$$

In triangle, OBC,

$$\tan \theta = 4/2 = 2$$

$$\text{and } \sin \theta = 2/\sqrt{5}$$

In triangle, BDC

$$\sin \theta = BD/2 \Rightarrow BD = 4/\sqrt{5}$$

$$\text{Length of chord of contact} = AB = 8/\sqrt{5}$$